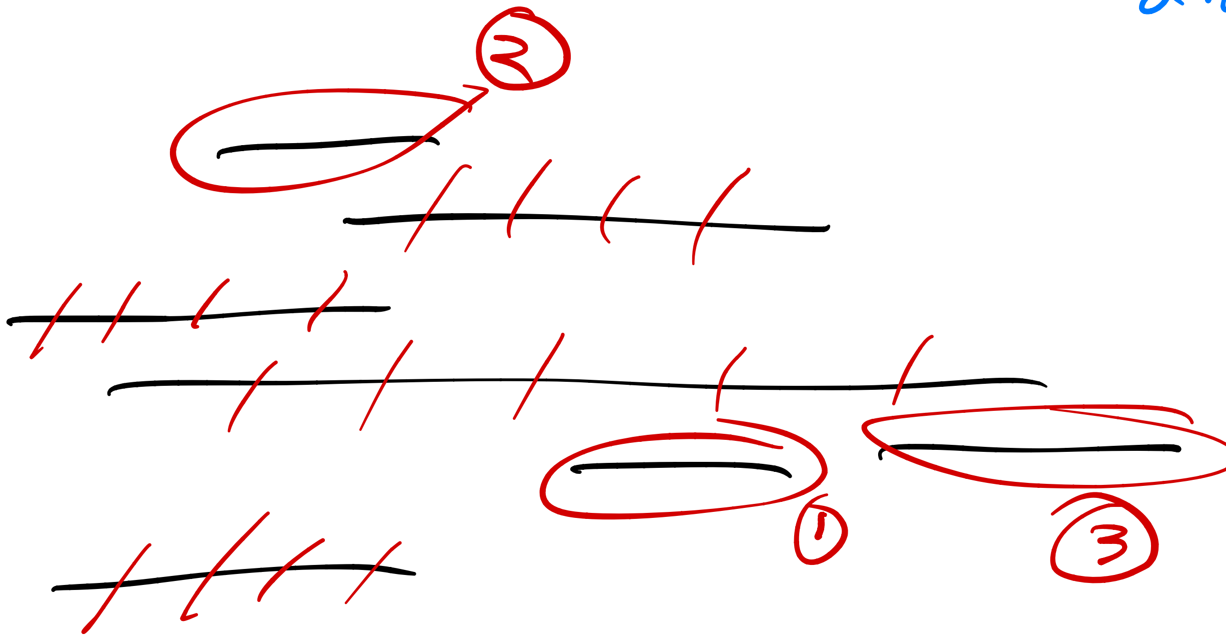
CS 252

W, 17 April 2024

R

Try "smallest interval available"



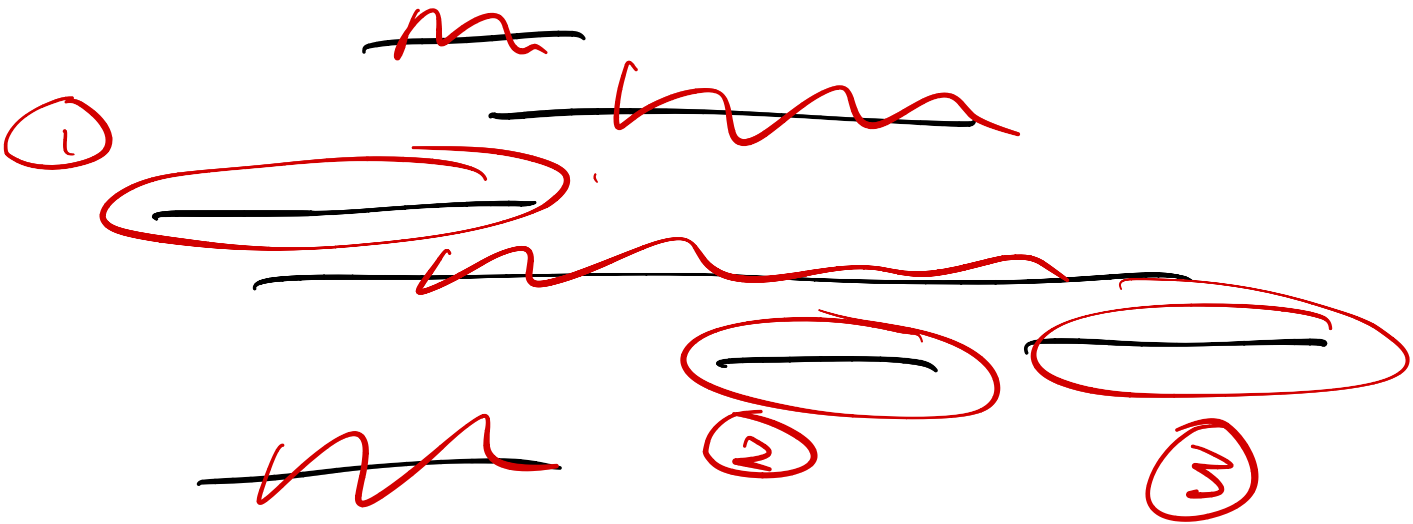
R

Why does "smallest first" fail?

counter-example

R

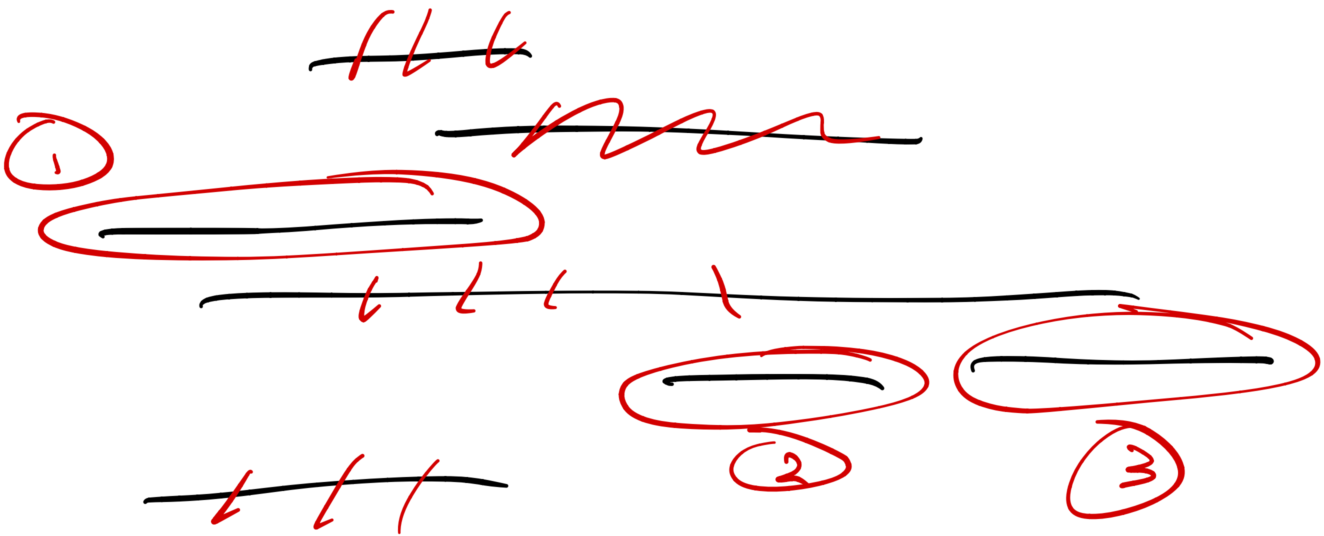
earliest start



Why does "earliest start" fail?

R

earliest finish



Thm. "Earliest finish" greedy alg. works for this problem.

Proof outline

Given Alg $\rightarrow A = \{i_1, \ldots, i_k\}$ ①

Suppose O is optimal $\rightarrow O = \{j_1, \ldots, j_m\}$ ②

m is as large as possible

prove $k \geq m$ ③

How many possible subsets of R are there?

$$2^{|R|}$$

Valid subsets (no overlap) $\leq 2^{|R|}$

finite set $R = \{a, b, c\}$

Dyson

$$2^3 \left\{ \begin{array}{ccc|c} 0 & 0 & 0 & \emptyset \\ 0 & 0 & 1 & \{c\} \\ & \vdots & & \\ 1 & 1 & 0 & \{a, b\} \\ & \vdots & & \\ 1 & 1 & 1 & \{a, b, c\} \end{array} \right.$$

Proof, continued

$A = \{i_1, \ldots, i_k\}$

$O = \{j_1, \ldots, j_m\}$

$f(i_1) \le s(i_2) \le f(i_2) \le s(i_3) \ldots$

$\text{\rotatebox{90}{$\le$}}$ (i.e. $f(i_1) \le f(j_1)$)

$f(j_1) \le s(j_2) \le \ldots$

① $f(i_1) \le f(j_1)$ Base

② $f(i_x) \le f(j_x)$ Induction

for $x = 2 \ldots k$

③ wrap it up