Layla Oesper, Jeff Ondich

One question of your choice (either #1 or #2 or #3) is due by **5:00PM Tuesday, April 23** and will be graded on completion only (1 point). Your full answers to all questions are due by **5:00PM Friday, April 26**. Each solution will be graded for correctness and clarity (4 points per question). Read the course information page/syllabus for further info about this 4-point scale.

0. Estimate the amount of time you spent on each question and include it at the top of your solution. Also, list your collaborators for each question at the top of your solution.

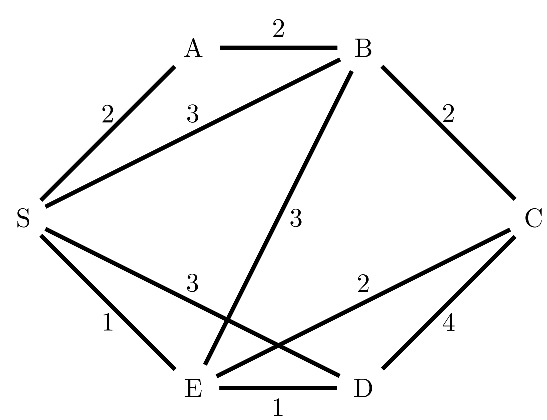
1. The State of Minnesota is rolling out a new program where they want to ensure that there is full cellular coverage along the entire length of Interstate-35 (here-forth called I-35) across the entire state. This way, when Layla's old and janky car breaks down, she can be sure to be able to call Jeff to come pick her up - no matter where on I-35 she is. To accomplish this goal, the State is going to have to put up a bunch of new cell towers along I-35 Unfortunately, they can't just put a cell tower wherever they want (some people don't want such things on their property). So the State has identified a number of candidate locations where they theoretically could put a cell tower. However, each cell tower costs a good amount of money to construct, so the State would like to accomplish this project using as few cell towers as possible. The final piece of the puzzle is that each cell tower only has a maximum usable range of 25 miles - meaning that a person must be within 25 miles of a cell tower - in any direction - in order to be able to use it. Your job is to help the State figure out where to put the cell towers (if there is a valid configuration that exists) so that the entirely of I-35 is covered by cell service.

   Formally, you can think of I-35 as a line segment $[0, L]$ of length L miles that has n possible cell tower locations (handed to you in an arbitrary order) where a cell tower could be built For simplicity, you can assume that a cell tower has zero width and each possible location is just a point on the line of length L. Give an algorithm that places as few cell towers as possible (each one must be placed at a valid location), while covering the entire length of I-35, or (correctly) reports that covering the entire highway is not possible given the set of n valid locations to place a cell tower.

   Describe a greedy algorithm to solve this problem, make a convincing argument (proof) that your algorithm correctly solves the desired problem and analyze its runtime.

2. (a) In order to run Dijkstra's algorithm on a graph G, what (if any) assumptions must you make about G? Explain (~1-2 sentences) why this assumption is necessary.

   (b) Run Dijkstra's algorithm on the graph below starting at node S. When you have choice of where to visit next, always pick the node that is first alphabetically. Report (1) the order you discover the nodes; (2) for each node v in the graph the shortest distance found from S; and (3) for each node v the sequence of nodes on the discovered shortest path from S to v.



   (c) Both Dijkstra's algorithm and BFS compute shortest paths. Explain when you should use each one over the other.

3. The custodians of Schiller have decided that Schiller needed his own Facebook account. He is now friends with a subset of Carleton students, who have friends of their own on Facebook, who also have friends on Facebook, etc. This has made me super exited because I want to get a message to Schiller and I think I can use my Facebook network to try and make that happen! In particular I want to find a sequence of people that connect me to Schiller via friendship links on Facebook and I will use those links to send a letter from person to person to eventually get the message to Schiller. For example, I send the letter to CS professor Eric Alexander, who will then send the letter to a friend from undergrad, who will then hand the letter to their cousin who is a current student, who will send the letter to Schiller (thus making my Schiller-friend number 3, and smaller than my Erdos number which is 4).

   Naturally, I want the letter to get to Schiller as fast as possible, with the fewest opportunities for trouble to arise. So I want to find a set of connected friendships between me and Schiller that is as short as possible. But, given the highly connected nature of the social structure surrounding Carleton students, alumni and faculty (did I mention that my best friend in graduate school was a Carleton alumna?), I suspect there are many such shortest routes from me to Schiller. So, my first task will be to actually identify **how many distinct shortest routes there are from me to Schiller** so that I can then identify the "best one" to use when sending my letter to Schiller. In order to minimize my delay in sending the invitation, I need a fast algorithm to solve this problem. This is where I need your help!

   More formally, here is the problem that I need your help to solve.

   Input: Undirected graph $G = (V, E)$; nodes $u, v \in V$.
   Output: The number of distinct shortest paths (shortest is determined in terms of number of edges traversed) from u to v. (To be clear, distinct just means any pair of such paths differ in at least one edge.)

   Provide a linear time algorithm (this can depend on both $|V|$ and $|E|$) to solve the above problem. Include a convincing argument (proof) that your algorithm is correct and analyze its running time.

   Hint 1: I'm only asking you to tell me how many of these paths there are. I am not asking you to find the paths. Make sure to keep this in mind! If you are actually computing the paths, you are probably doing too much work.

   Hint 2: I suggest thinking about how you might modify an algorithm we already know about to help you solve this problem.