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Due date There is no checkpoint due date for this assignment. It is all due at 5:00PM Friday, April 19. Problem 1 is worth 4 points, graded on the usual scale. Problem 2 is worth 5 points–1 point per item.

0. Estimate the amount of time you spent on each question and include it at the top of your solution. Also, list your collaborators and resources used for each question at the top of your solution.

1. The local authorities have been investigating a number of incidents where customer private information has been stolen and transferred in-between various individuals that have appeared to use the information nefariously. They have been studying this network of n corrupt individuals for some time now and have assembled what you can view as an n-by-n adjacency matrix representing a *directed* graph of data exchanges, where an edge $u \to v$ means that on at least one occasion u transferred stolen information to v. The chief investigator wants to identify the *central bigwig* in the scheme – the person who has received information from all $n-1$ other people and who has not given any data to anyone else. (In graph-theoretic terminology, a central bigwig is a node with indegree $n-1$ and outdegree zero.) The chief investigator wants your help before even more private data is compromised.

   When a graph is represented using an adjacency matrix, nearly all graph algorithms require $\Omega(n^2)$ time on an n-node graph: you usually have to look at every entry in the adjacency matrix so that you can at least identify which edges are in the graph. Identifying a central bigwig in $\Theta(n^2)$ time is straightforward. But the chief investigator is in a time crunch and needs something faster. Give an algorithm that in $\Theta(n)$ time identifies a central bigwig in a given graph (given as an adjacency matrix) if it has one, or correctly reports that there is no central bigwig.

2. **Graph examples**

   For each of the following, provide examples of graphs having the specified properties. Include a brief explanation of why each of your examples satisfies the expected constraints. Use the textbook's notation by naming your graphs $G = (V, E)$ and letting $n = |V|$ and $m = |E|$.

   Please show your examples as images rather than lists of edges. Look at latex-miscellany.pdf and latex-miscellany.tex for some help drawing graphs and including images in LaTeX documents. Your graphs may appear in either form–that is, the graph-drawing LaTeX module is great, but if it's easier for you, go ahead and draw a graph by hand and include a well-lighted image of it in your PDF file.

   (a) Show an undirected graph that is not a tree, but for which $m = n - 1$.

   (b) Show a directed graph that is weakly connected but not strongly connected. A directed graph is *weakly connected* if replacing all directed edges with undirected edges produces a connected (undirected) graph.

   (c) Show an undirected graph that has a Hamiltonian cycle but not an Eulerian cycle

   (d) Show a a connected weighted graph and two nodes u and v for which there is no lowest-weight path from u to v. (Note that having non-unique lowest-weight paths–i.e. two or more paths from u to v that have the identical, lowest weight is not what we're after here. We're looking for a situation where there are zero paths that can qualify as having the lowest weight, which is weirder than "non-unique lowest-weight path".)

   (e) How different can the BFS and DFS trees of a given graph be? Here's one approach to this question. Let the diameter of a graph be the longest number of edges in the shortest path between two nodes in the graph. (This is sometimes called the "length of the longest shortest path" in the graph.) For this last item, show: a graph G, a node s, a BFS tree T_B, and a DFS tree T_D such that the diameter of T_D is at least 2 times as large as the diameter of T_B. Then, show how you would extend your example to make the factor 3 or 10 or 1000 instead of 2.