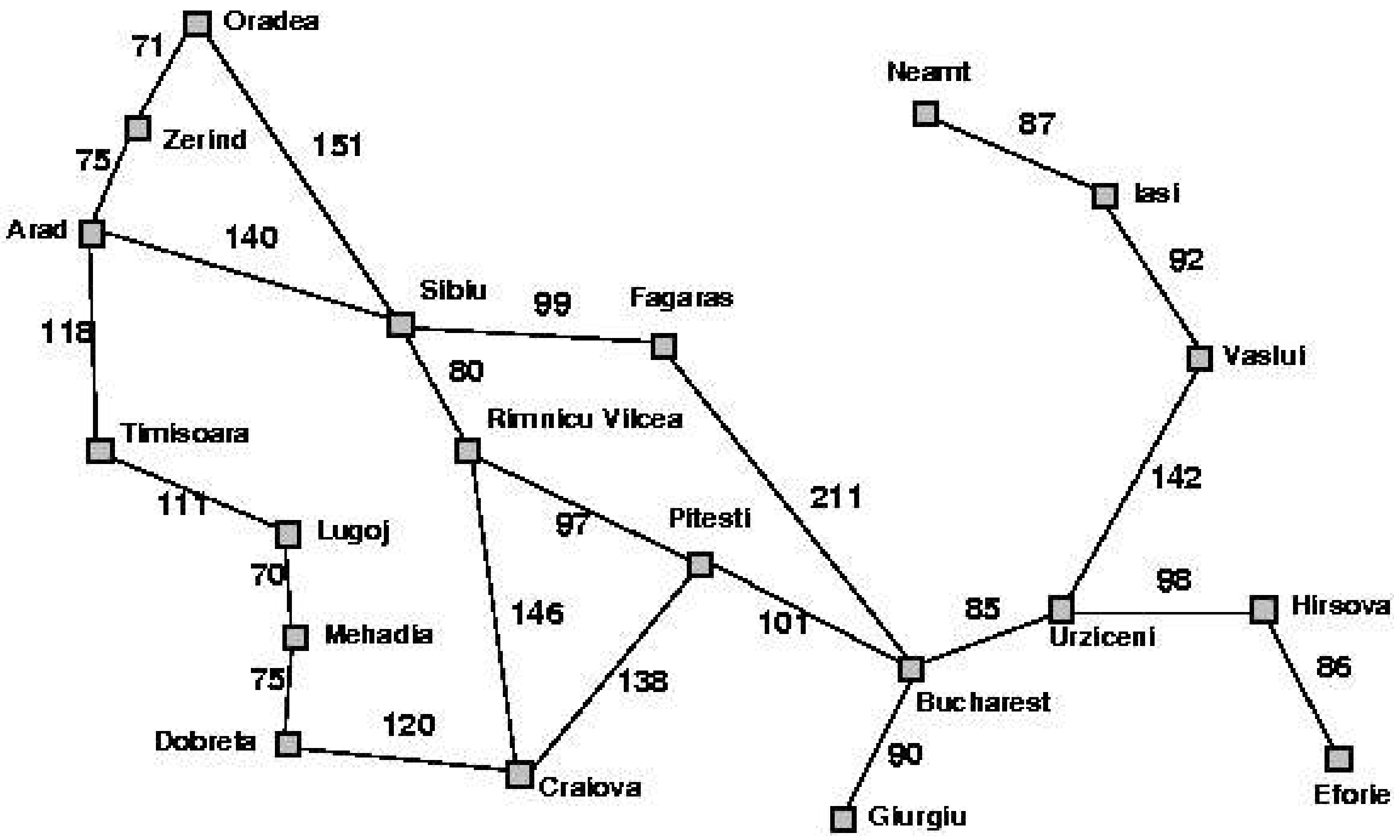
Informed Search Methods

- How can we improve searching strategy by using intelligence?
- Map example:
  - Heuristic: Expand those nodes closest in "as the crow flies" distance to goal
- 8-puzzle:
  - Heuristic: Expand those nodes with the most tiles in place
- Intelligence lies in choice of heuristic

Best-First Search

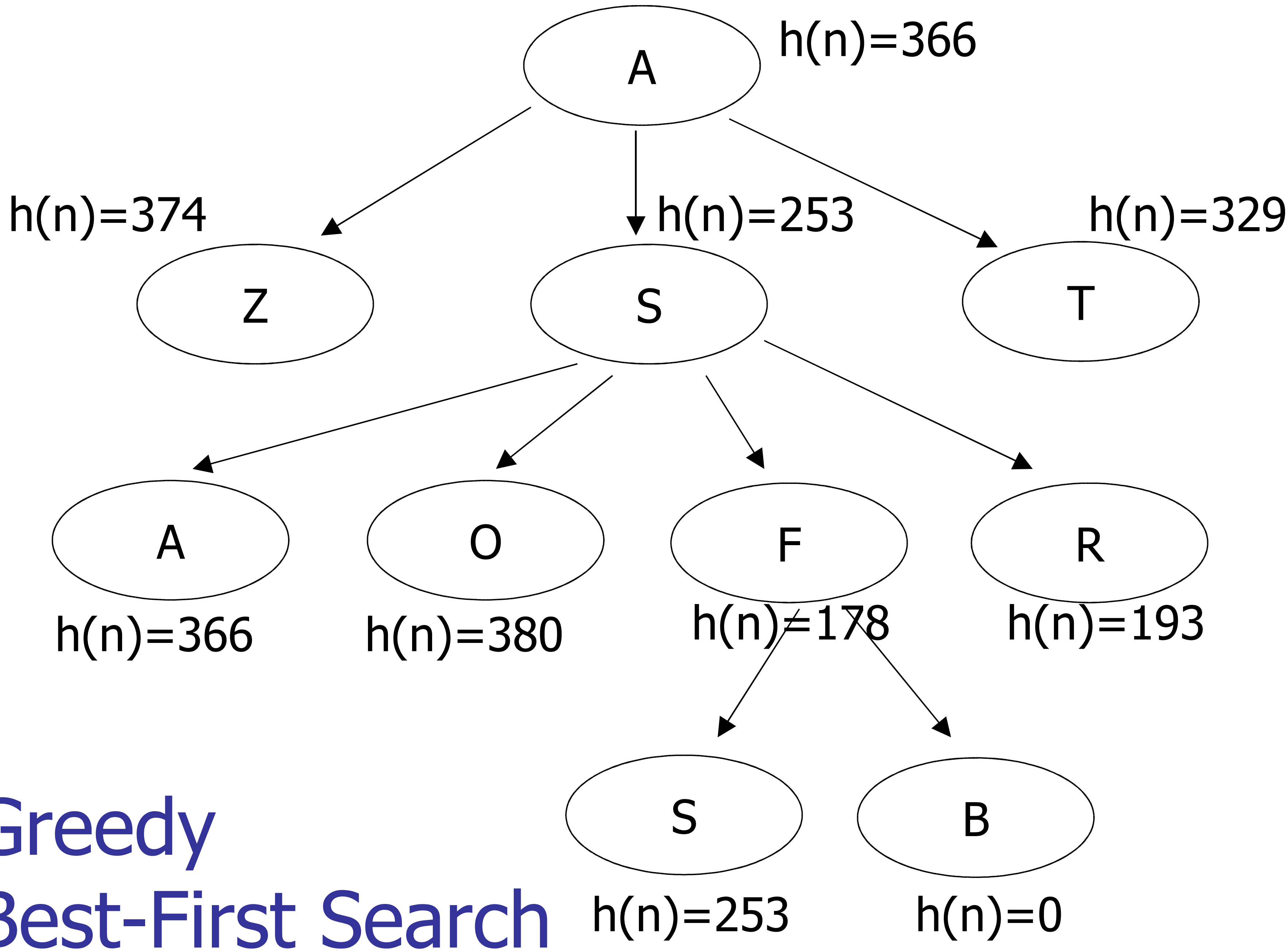
- Create evaluation function f(n) which returns estimated "value" of expanding node
- Example: Greedy best-first search
  - "Greedy": estimate cost of cheapest path from node n to goal
  - h(n) = "as the crow flies distance"
  - f(n) = h(n)

Romania with step costs in km



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374



Greedy Best-First Search

Greedy Best-First Search

- Expand the node with smallest h
- Why is it called greedy?
  - Expands node that appears closest to goal
- Similar to depth-first search
  - Follows single path all the way to goal, backs up when dead end
- Worst case time:
  - $O(b^m)$, m = depth of search space
- Worst case memory:
  - $O(b^m)$, needs to store all nodes in memory to see which one to expand next

Greedy Best-First Search

- Complete and/or optimal?
  - No – same problems as depth first search
  - Can get lost down an incorrect path
- How can you (help) to prevent it from getting lost?
  - Look at shortest total path, not just path to goal

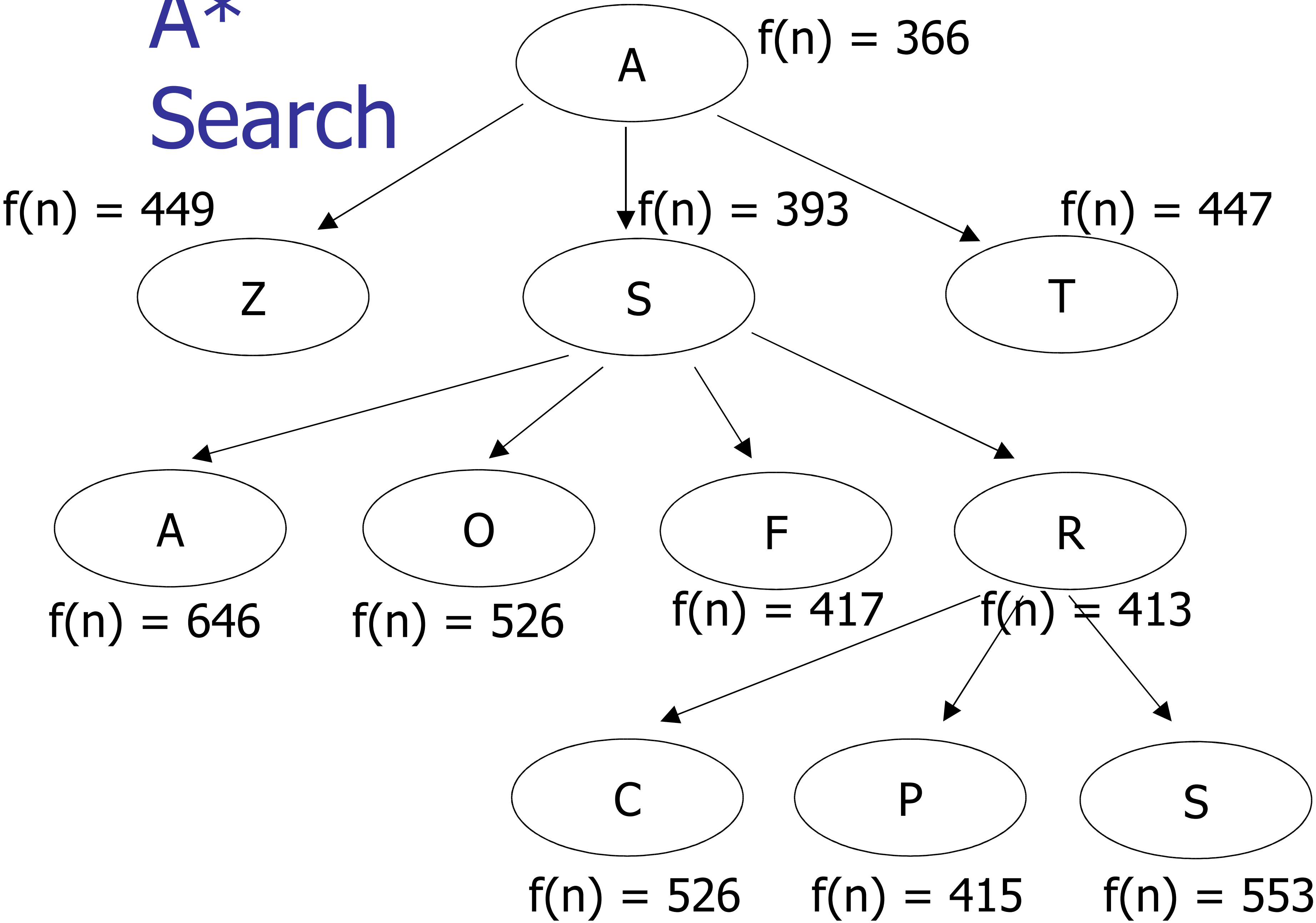
A* search (another Best-First Search)

- Greedy best-first search minimizes
  - h(n) = estimated cost to goal
- Uniform cost search minimizes
  - g(n) = cost to node n
  - Example of each on map
- A* search minimizes
  - f(n) = g(n) + h(n)
  - f(n) = best estimate of cost for complete solution through n

A* search

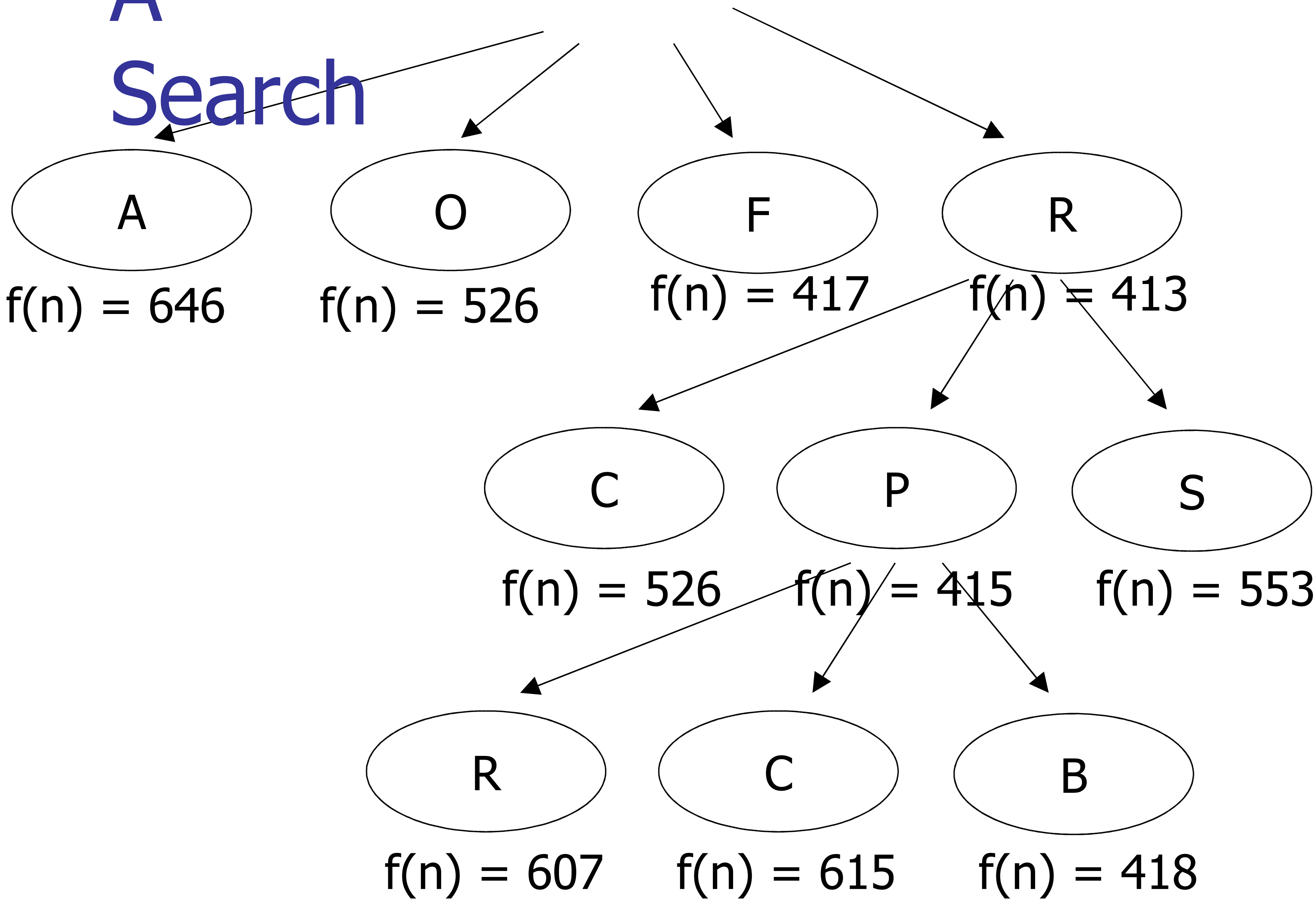
- Under certain conditions:
  - Complete
  - Terminates to produce best solution
- Conditions
  - (assuming we don’t throw away duplicates)
  - h(n) must never overestimate cost to goal
    - admissible heuristic
    - “optimistic”
    - “Crow flies” heuristic is admissible

A* Search

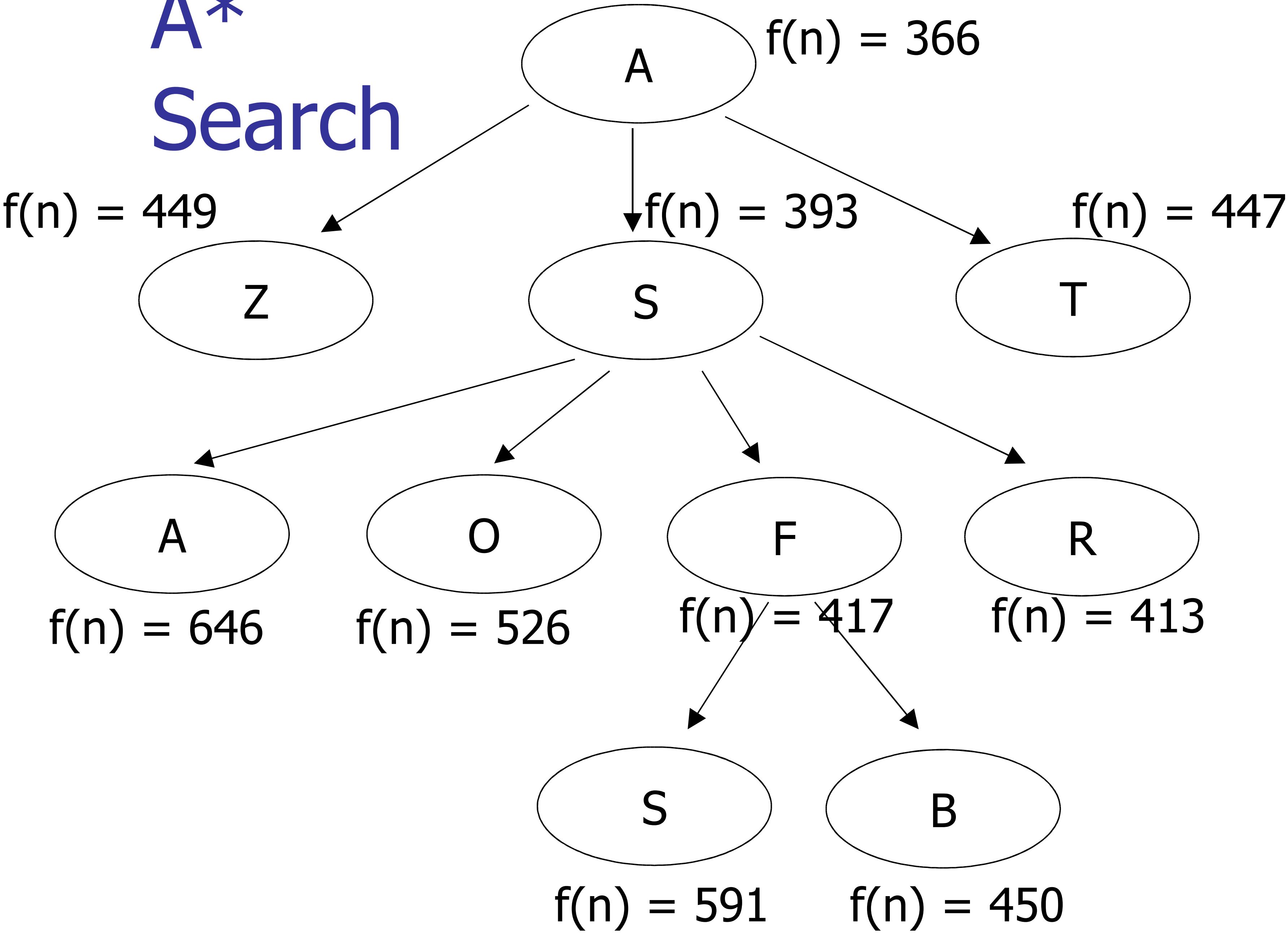


A
f(n) = 366
f(n) = 449
Z
f(n) = 393
S
f(n) = 447
T
A
f(n) = 646
O
f(n) = 526
F
f(n) = 417
R
f(n) = 413
C
f(n) = 526
P
f(n) = 415
S
f(n) = 553

A* Search


A
f(n) = 646
O
f(n) = 526
F
f(n) = 417
R
f(n) = 413
C
f(n) = 526
P
f(n) = 415
S
f(n) = 553
R
f(n) = 607
C
f(n) = 615
B
f(n) = 418

A* Search


A
f(n) = 366
Z
f(n) = 449
S
f(n) = 393
T
f(n) = 447
A
f(n) = 646
O
f(n) = 526
F
f(n) = 417
R
f(n) = 413
S
f(n) = 591
B
f(n) = 450

A* terminates with optimal solution

- Stop A* when you try to expand a goal state.
  - This the best solution you can find.
- How do we know that we're done when the next state to expand is a goal?
  - A* always expands node with smallest f
  - At a goal state, f is exact.
  - Since heuristic is admissible, f is an underestimate at any non-goal state.
  - If there is a better goal state available, with a smaller f, there must be a node on graph with smaller f than that – so you would be expanding that instead!

More about A*

- Completeness
  - A* expands nodes in order of increasing f
  - Must find goal state unless
    - infinitely many nodes with f(n) < f*
      - infinite branching factor OR
      - finite path cost with infinite nodes on it
- Complexity
  - Time: Depends on h, can be exponential
  - Memory: $O(b^m)$, stores all nodes

Valuing heuristics

- Example: 8-puzzle
  - h1 = # of tiles in wrong position
  - h2 = sum of distances of tiles from goal position (1-norm, also known as Manhattan distance)
- Which heuristic is better for A*?

Which heuristic is better?

- h2(n) >= h1(n) for any n
  - h2 dominates h1
- A* will generally expand fewer nodes with h2 than with h1
  - All nodes with f(n) < C* (cost to best solution) are expanded.
  - Since h2 >= h1, any node that A* expands with h2 would also be expanded with h1
  - But A* may be able to avoid expanding some nodes with h2 (larger than C*)
  - (Exception where you might expand a state with h2 but not with h1: if f(n) = C*).
- Better to use larger heuristic (if not overestimate)

Inventing heuristics

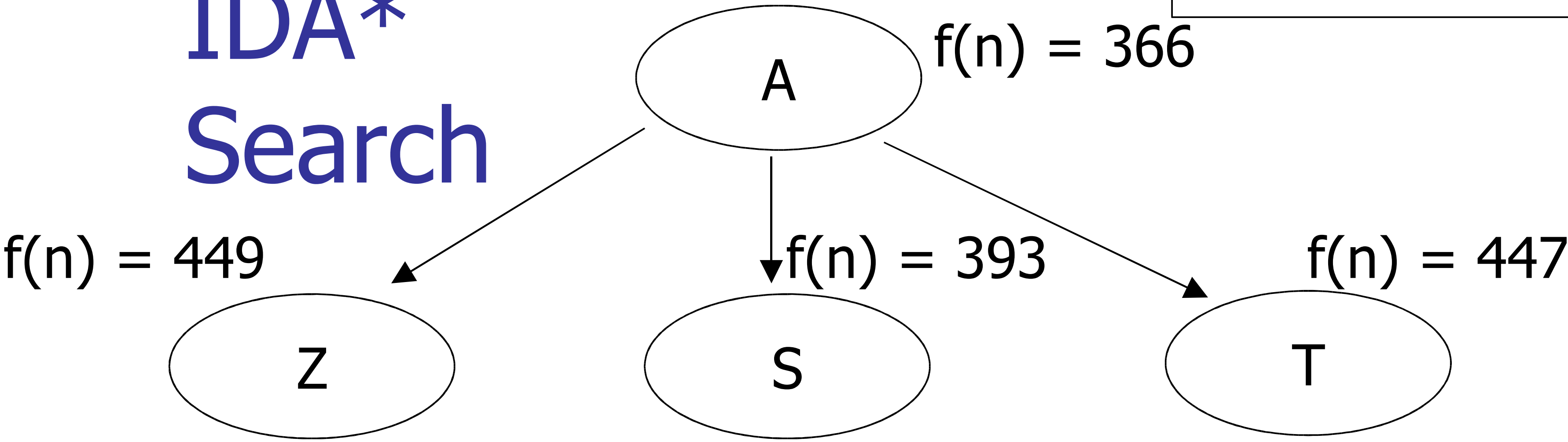
- h1 and h2 are exact path lengths for simpler problems
  - h1 = path length if you could transport each tile to right position
  - h2 = path length if you could just move each tile to right position, irrelevant of blank space
- Relaxed problem: less restrictive problem than original
- Can generate heuristics as exact cost estimates to relaxed problems

Memory Bounded Search

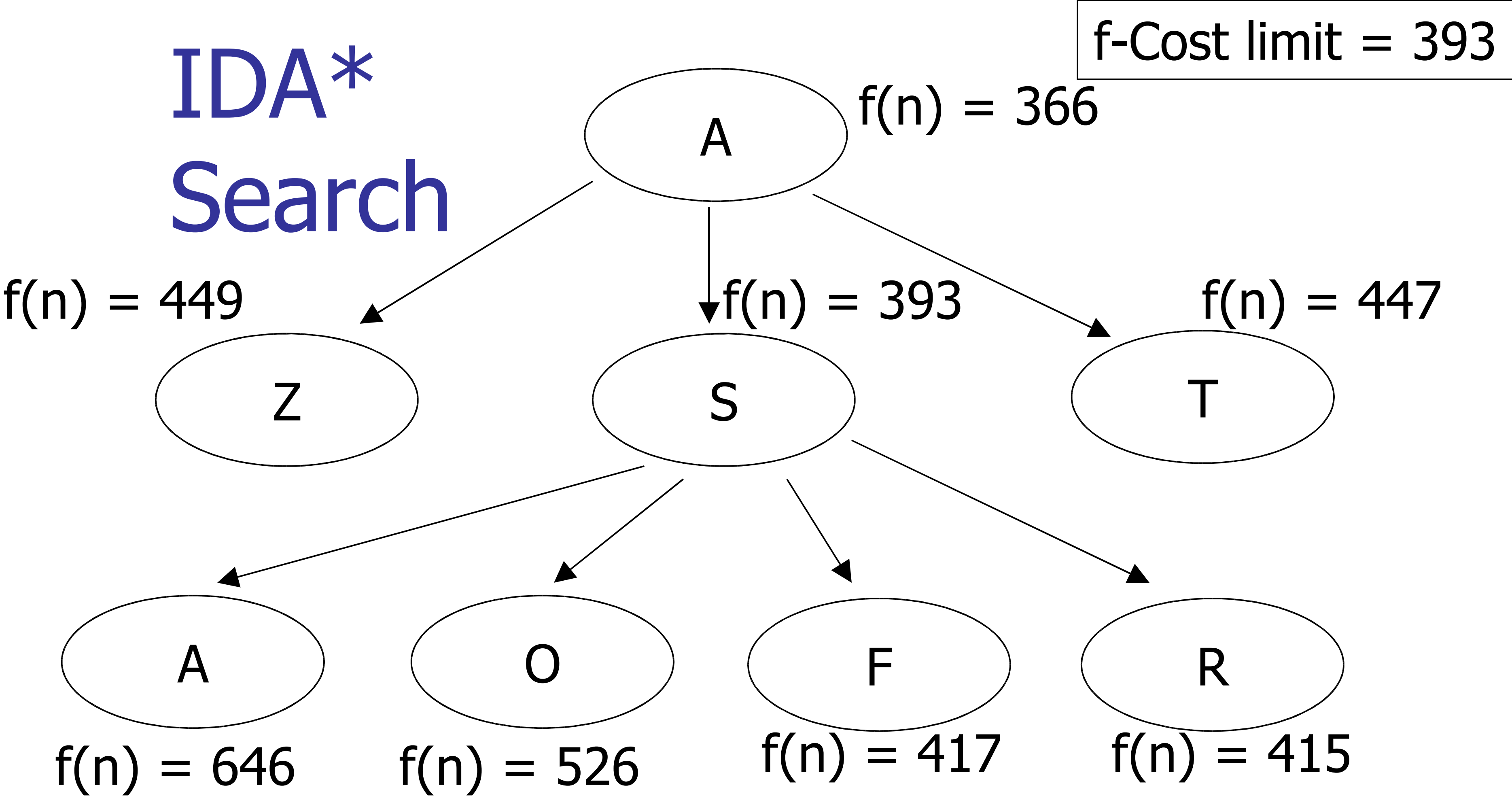
- Can A* be improved to use less memory?
- Iterative deepening A* search (IDA*)
  - Each iteration is a depth-first search, just like regular iterative deepening
  - Each iteration is not an A* iteration: otherwise, still $O(b^m)$ memory
  - Use limit on cost (f), instead of depth limit as in regular iterative deepening

IDA* Search

f-Cost limit = 366



IDA* Search


f-Cost limit = 393
A
f(n) = 366
f(n) = 449
Z
f(n) = 393
S
f(n) = 447
T
A
f(n) = 646
O
f(n) = 526
F
f(n) = 417
R
f(n) = 415

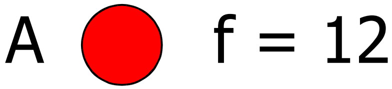
IDA* Analysis

- Time complexity
  - If cost value for each node is distinct, only adds one state per iteration
    - BAD!
    - Can improve by increasing cost limit by a fixed amount each time
  - If only a few choices (like 8-puzzle) for cost, works really well
- Memory complexity
  - Approximately O(bd) (like depth-first)
- Completeness and optimality same as A*

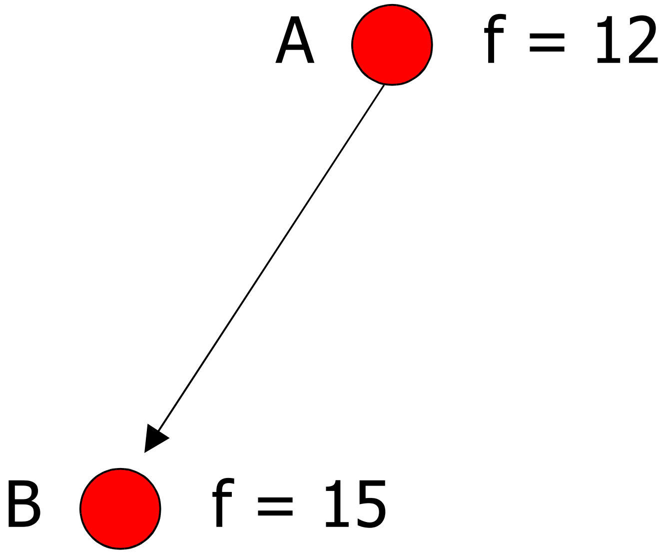
Simplified Memory-Bounded A* (SMA*)

- Uses all available memory
- Basic idea:
  - Do A* until you run out of memory
  - Throw away node with highest f cost
    - Store f-cost in ancestor node
    - Expand node again if all other nodes in memory are worse

SMA* Example: Memory of size 3

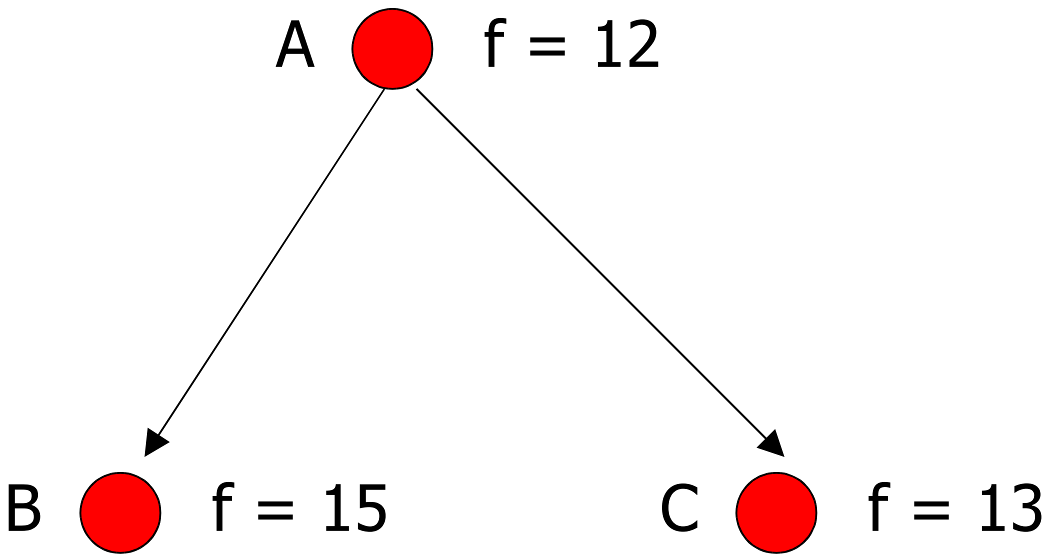


SMA* Example: Memory of size 3



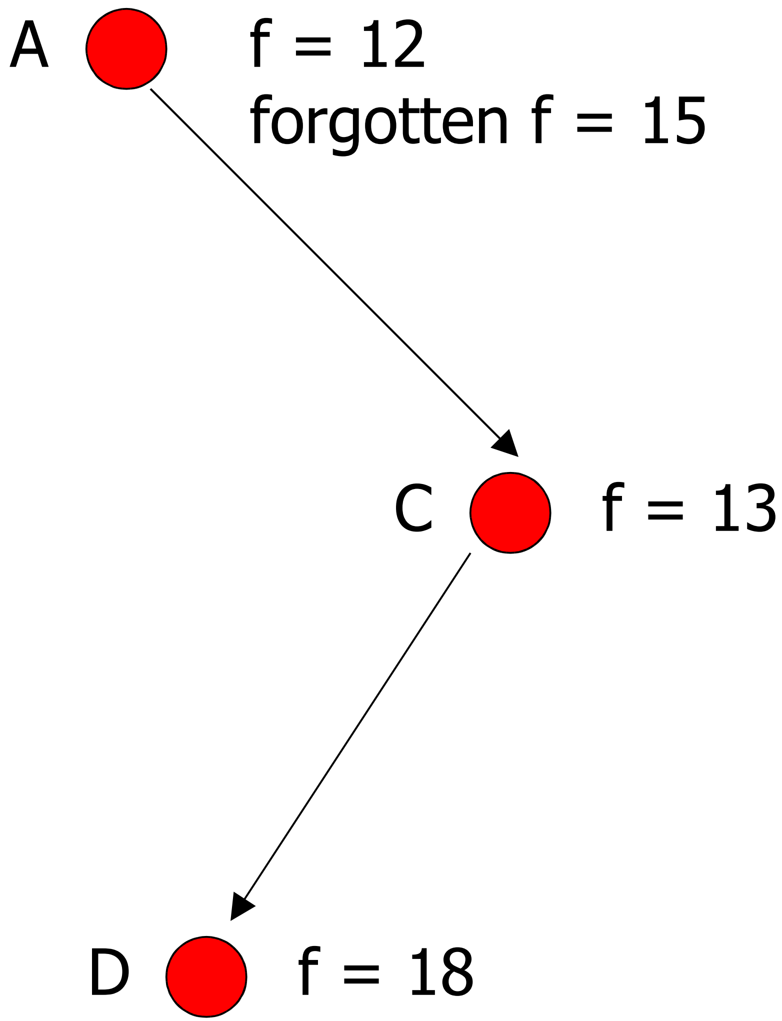
Expand to the left

SMA* Example: Memory of size 3



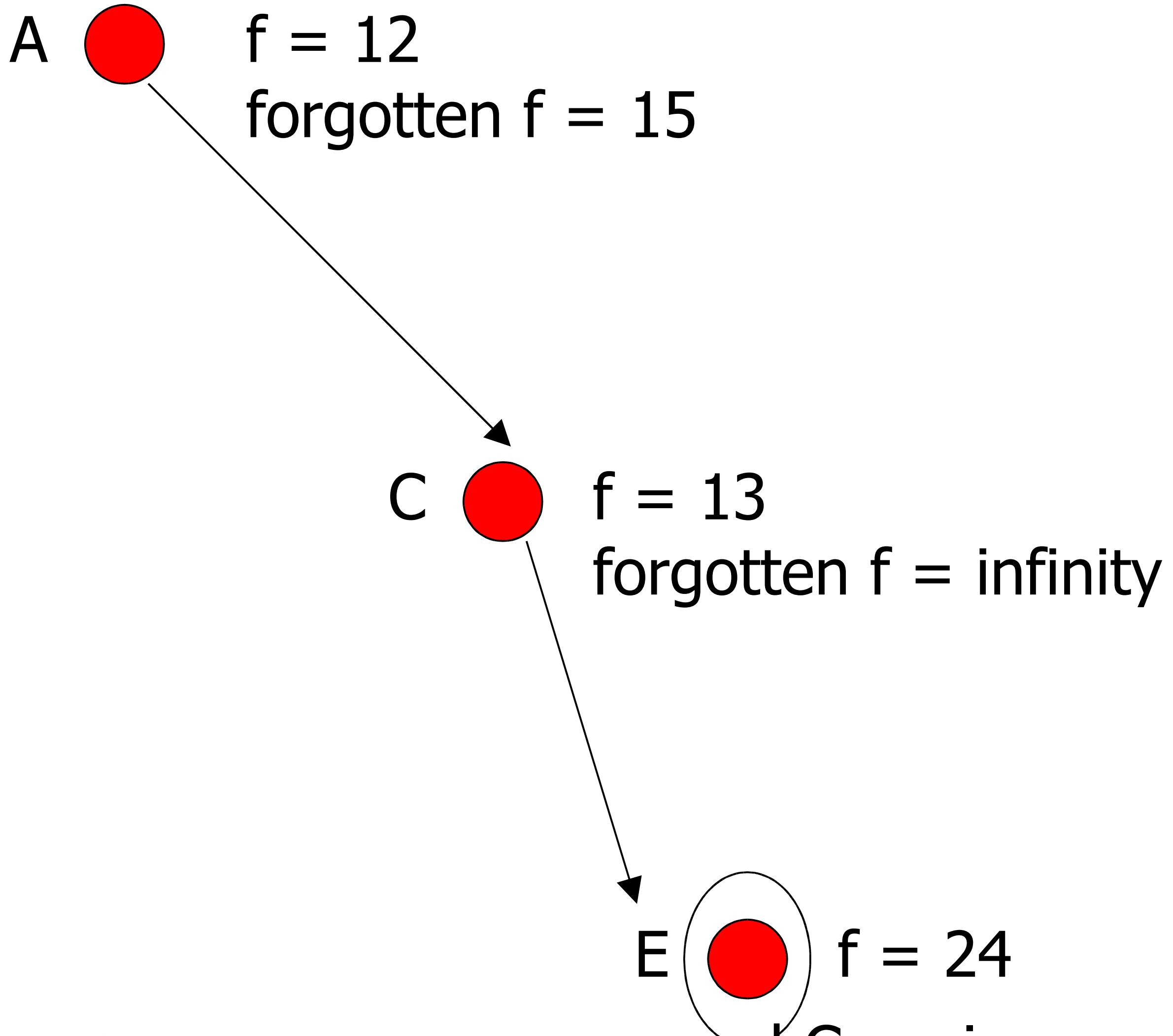
Expand node A, since f smaller

SMA* Example: Memory of size 3



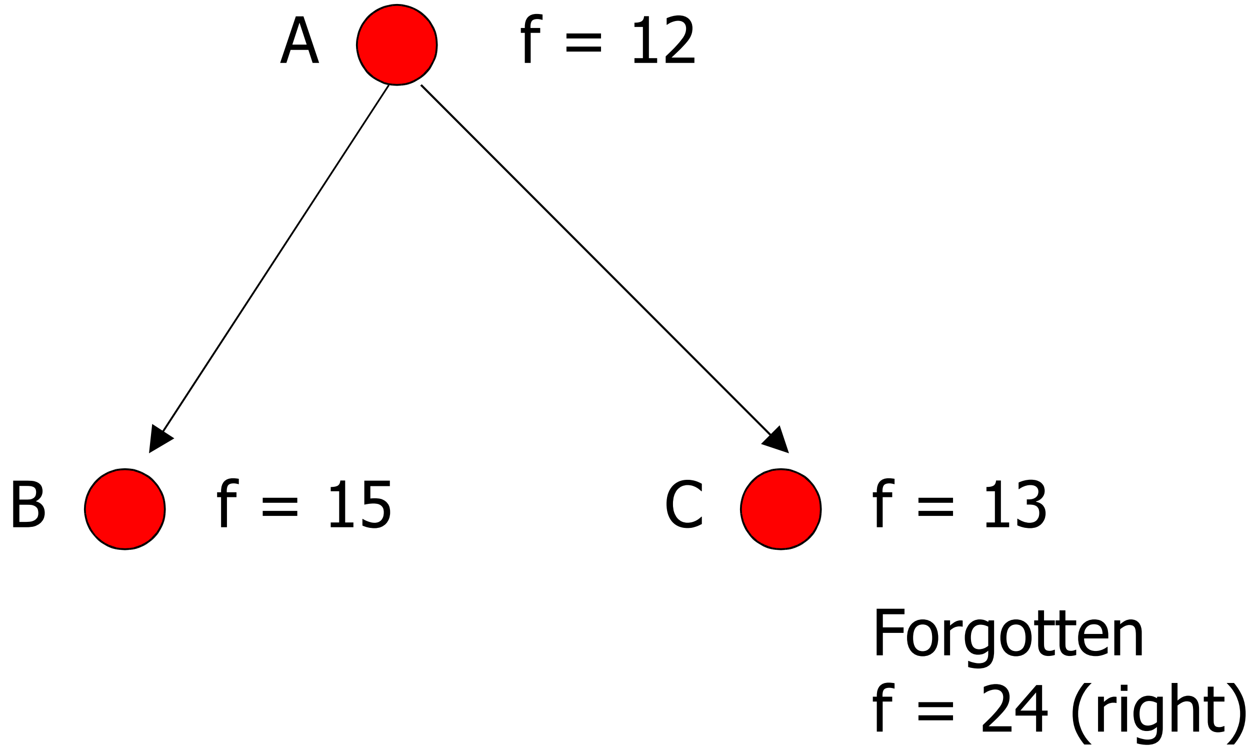
Expand node C, since f smaller

SMA* Example: Memory of size 3



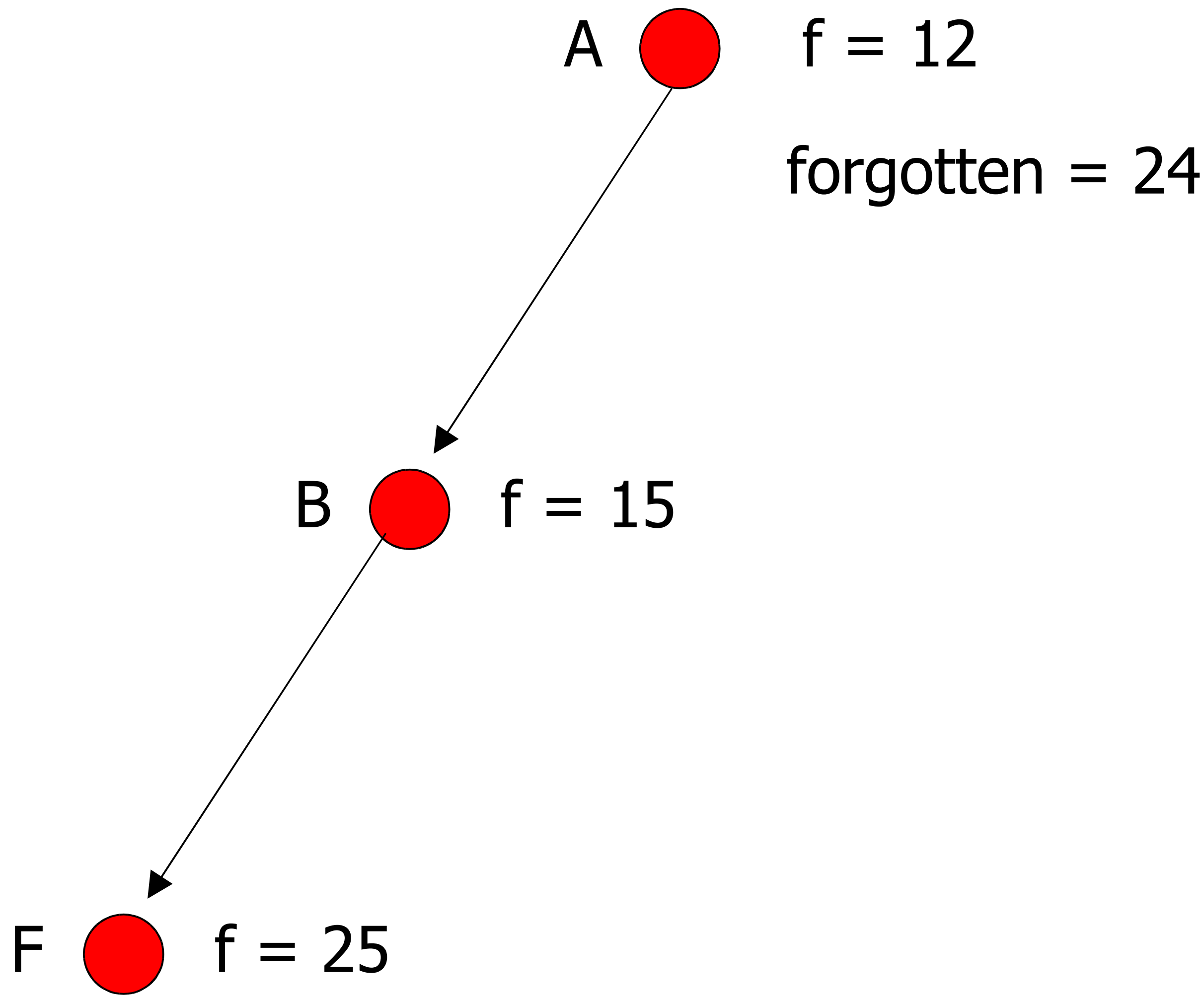
Node D not a solution, no more memory: so expand C again

SMA* Example: Memory of size 3



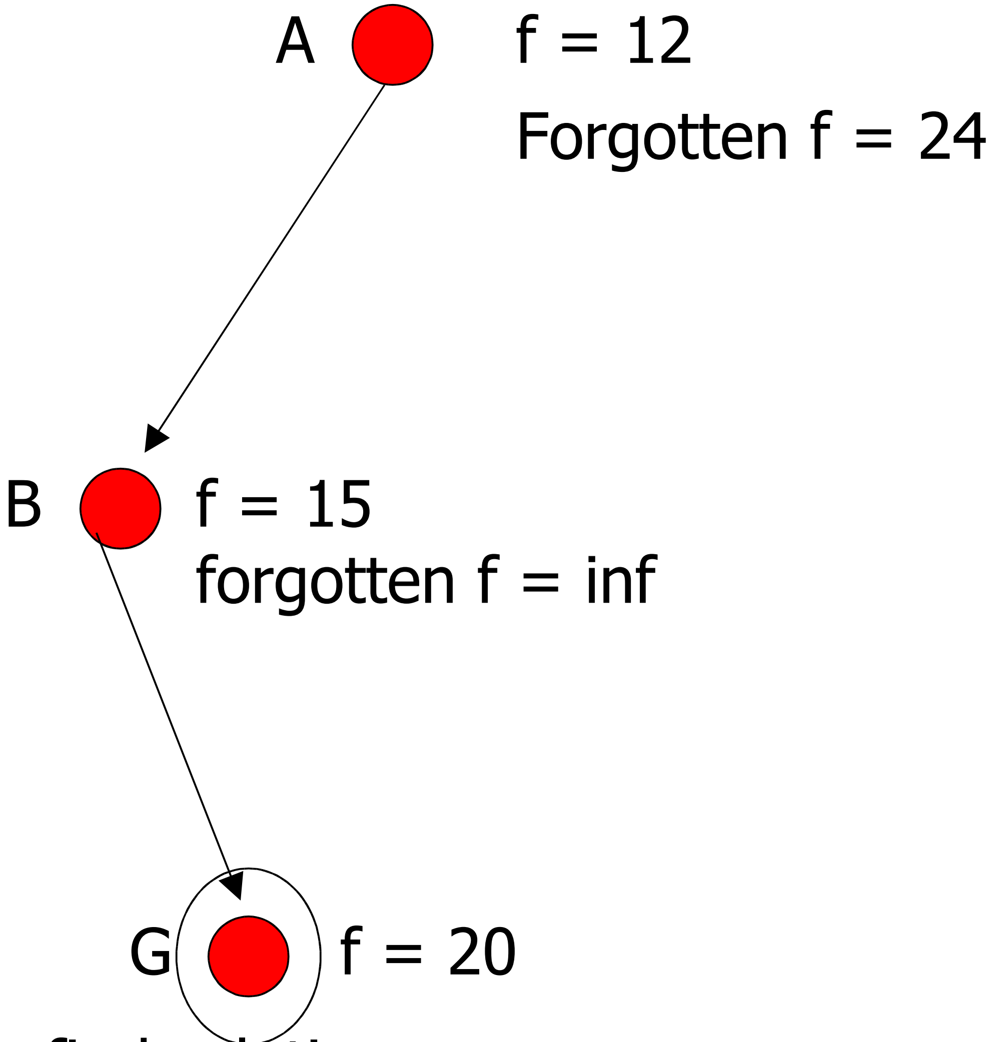
Re-expand A; record new f for C

SMA* Example: Memory of size 3



Expand left B: not a solution, so useless

SMA* Example: Memory of size 3



Expand right B: find solution

SMA* Properties

- Complete if can store at least one solution path in memory
- Finds best solution (and recognizes it) if path can be stored in memory
    - Otherwise, finds best that can fit in memory