Math 5707 Exam 2

Due at the beginning of lecture on Wednesday, April 16, 2014.

Please staple this sheet to the front of your solutions.

There are 6 problems, each worth 7 points. **Turn in solutions for (at most) 5 of them.** If you turn in work for all 6 problems, an arbitrary subset of 5 problems will be graded. Be sure to justify all your work: answers without sufficient justification will receive no credit.

You may use resources such as books and the Internet. Do not collaborate or consult human sources besides the instructor. Clearly indicate any outside sources consulted, and make sure to understand the solutions sufficiently to *explain them in your own words*. Solutions which the instructor views as insignificant alterations of outside sources will receive no credit.

Name: ______________________________

ID: ______________________________

Problem 1 (7 points)	
Problem 2 (7 points)	
Problem 3 (7 points)	
Problem 4 (7 points)	
Problem 5 (7 points)	
Problem 6 (7 points)	
$\sum$ (35 points total)	

Problem 1. Given a graph $G = (V, E)$ with $\delta(G) \geq 2$, prove that there is a connected graph H on the same vertex set V such that $d_G(v) = d_H(v)$ for all $v \in V$.

Problem 2. Let G be a planar graph on n vertices. Suppose k is the length of a shortest cycle in G. Prove that G has at most $(n-2)\frac{k}{k-2}$ edges.

Problem 3. For $k, \ell \in \mathbb{N}$ such that $1 \leq k \leq \ell$, prove that there is a graph G with connectivity $\kappa(G) = k$ and edge-connectivity $\lambda(G) = \ell$.

[**Hint:** See Section 1.4 in Diestel for the definition of edge-connectivity $\lambda(G)$, and note that Proposition 1.4.2 explains why $k \leq \ell$ is assumed.]

Problem 4. Let $G = (V, E)$ be a plane graph whose vertices are all on the boundary of the outer face. Prove that there is a partition of V into two sets V_1 and V_2 such that each induced subgraph $G[V_1]$ and $G[V_2]$ is a disjoint union of paths.

[**Hint:** Consider the parity of the distance $d(x, y)$, defined in Section 1.3 of Diestel, from a fixed vertex x.]

Problem 5. For $n \in \mathbb{N}$, prove that there exists a bipartite, 3-regular, planar graph with $2n$ vertices if and only if $n \geq 4$ and $n \neq 5$.

Problem 6. For $k \in \mathbb{N}$, let $G = (V, E)$ be a k-connected graph. Suppose $f : V \to \mathbb{Z}$ is a function with integer values such that $\sum_{v \in V} f(v) = 0$ and $\sum_{v \in V} |f(v)| = 2k$, where $|x|$ is the absolute value of x. Prove that there are k independent paths such that $|f(v)|$ of them have v as an end for each $v \in V$.