Math 4990 Problem Set 5

Partial solutions and comments

Problem	Points	Mean
Problem 1	2	1.2
Problem 2	2	1.6
Problem 3	2	1.6
Problem 5	2	1.4
Problem 6	2	0.7
$\sum$	10	6.3

Problem 1. Prove that the convex hull $\mathrm{conv}(S)$ of any set S is convex.

Some of you proved that intersection of two convex sets is convex. By induction, this shows that the intersection of a *finite* collection of convex sets is convex. This does not solve the problem as we have an *infinite* collection of convex sets to intersect. You lost a point if you tried doing this.

Also, it was explicitly stated not to assume S lies in a plane, so you should not be referencing polygons.

Problem 5. Let $P_1, \dots, P_n \subset \mathbb{R}^2$ be rectangles whose sides are parallel to the x- and y-axes. Show that if every *two* of them intersect then they all intersect, *i.e.*, there is a point $z \in P_1, \dots, P_n$.

You should consider using Helly theorem (possibly multiple times), as opposed to proving it from the ground up.

Problem 6. Recall this corollary of Helly theorem, stated in class:

> Let $A \subset \mathbb{R}^2$ be a fixed convex set and let $X_1, \dots, X_n \subset \mathbb{R}^2$ be convex sets such that every three of them intersect a translation of A. There exists a translation of A that intersects all sets X_i.

For each i, let $Y_i = \{y \in \mathbb{R}^2 : (A + y) \cap X_i \neq \varnothing\}$, where $A + y := \{a + y : a \in A\}$ is the translation of A by y. In order to apply Helly theorem to obtain the corollary, show that the Y_i are convex.

The most common mistake is to claim that $Y_i = (A + y) \cap X_i$ and conclude from the (correct) fact that the intersection of two convex sets is convex. Indeed, Y_i is the collection of ways to translate A such that it intersects X_i.