# Math 4990 Problem Set 11

*Due Tuesday, Dec 01, 2015 in class*

Please refer to previous problem sets for instructions, including but not limited to the collaboration policy.

The Fair Division lecture is based on Chapter 4 of [P], which you are welcome to peruse.

There is no class on Nov 24; try dividing turkey fairly between family members.

## Assignment

**Problem 1.** Specify some reasonable definition of general position and prove the following:

> Let $X \subset \mathbb{R}^2$ be a set of $n = 6k$ points in general position. There exist three concurrent lines separating $X$ into six groups of $k$ points each.

Remember that the Intermediate Value Theorem applies only to *continuous* functions. Recall that in class, we already saw that six collinear points cannot be partitioned this way. As such, you certainly need to use the general position assumption somewhere!

**Problem 2.** Rays emanating from a common point are **equispaced** if the angles between adjacent rays are all the same. Prove or disprove:

> Every convex polygon $P \subset \mathbb{R}^2$ admits an equipartition into five parts by five equispaced rays.

**Problem 3.** We say $f$ is **crazy** if $f : [0,1] \to \mathbb{R}$ is a continuous function with $f(0) = f(1) = 0$. The interval $[x, y]$ is a **(horizontal) chord** of length $y - x$ if $f(x) = f(y)$. Let

$$L(f) = \{y - x : f(x) = f(y),\ 0 \le x \le y \le 1\}$$

denote the set of lengths of horizontal chords.

Recall that $\frac{1}{n} \in L(f)$ for any crazy $f$ and $n \in \mathbb{N}$. Prove that this list of lengths is best possible; *i.e.*, prove:

> For every $a \in (0,1) \setminus \{\frac{1}{n} : n \in \mathbb{N}\}$, there exists a crazy $f$ so that $a \notin L(f)$.

**Problem 4.** Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Suppose it is **periodic**, *i.e.*, there exists $t > 0$ (called the period) such that $f(x) = f(x+t)$ for all $x$. Show that $f$ has horizontal chords of any length.