Math 4990 Problem Set 7

Due Tuesday, Oct 27, 2015 in class

Please refer to previous problem sets for instructions, including but not limited to the collaboration policy.

Errata

p.68, last line, "by our induction hypothesis establishes the theorem."

p.102, Theorem 4.9, "if and only if for each point x in e, there exists a circle centered at x with two sites on its boundary and none in its interior."

Assignment

Liberally peruse **pages 66–69, 98–102** of [DO].

[DO] Exercises 3.19, 3.20 (for $n \geq 4$), 4.4, and 4.5 ("simple" means at most a few sentences).

Problem 5. Let G be a graph that is maximally planar with at least 4 vertices. Suppose vertices a, b, c are pairwise joined by edges. Show that G has a vertex v distinct from a, b, c such that the degree of v is at most five.

Note that this is a strengthening of Exercise 3.14 we used in class for the proof of Fáry theorem.

Problem 6. Recall that the number of triangulations of a convex $(n+2)$-gon is the Catalan number C_n. For infinitely many values of n, construct two sets $S, S' \subset \mathbb{R}^2$ each with $n+2$ points such that the number of triangulations of S is greater than C_n and the number of triangulations of S' is nonzero but less than C_n. (See Exercises 3.15 and 3.18.)

Note that "for infinitely many values of n" is a phrase mathematicians use when they want something more general than $n = 23$, say, but do not need it for every single value of n. For example, perhaps your construction works only for even n greater than 42, prime numbers, or n such that its proper positive integer divisors sum to itself. We refer to these as "infinite families of counterexamples."