Math 4990 Problem Set 4

Due Tuesday, Oct 6, 2015 in class

Please refer to previous problem sets for instructions, including but not limited to the collaboration policy.

Assignment

Liberally peruse **pages 20–27** of [DO]. See errata below.

You may use Dehn–Hadwiger theorem, which we proved, and Sydler theorem, which we will not prove. Think carefully and make sure to cite the correct one! You may use the notation $P \sim Q$ to denote scissors congruence. The symbol is `\sim` in LaTeX.

[DO] Exercises 1.47, 1.51, and 1.65. Please refrain from looking at solutions for 1.65 (or any other problem).

Problem 4. Two polygons are **∂-congruent** if one can be decomposed into polygonal pieces and rearranged to form the other, such that the boundary points remain on the boundary. Prove that polygons of the same area and perimeter are ∂-congruent.

Problem 5. A polygon is **centrally symmetric** if it is invariant under a rotation by π (or equivalently, reflection through a point). Two polygons are **T-congruent** if one can be decomposed into polygonal pieces and rearranged *by translations alone* to form the other; *i.e.*, rotations are not allowed.

Prove or disprove: Centrally symmetric polygons with equal area are T-congruent.

Problem 6. Take an n-gon, a translated copy of it (not in the same plane), and join corresponding vertices by edges to form n new faces (necessarily parallelograms). This polyhedron with $n + 2$ faces is called a **prism**. Prove that prisms are scissors congruent to cubes (of the same volume). Do not assume that the prisms are *right* prisms, *i.e.*, they might be slanted.

Errata

Problem 7. Exercise 1.48 is <u>false</u>. Optionally, for no credit, provide a counter-example.

The proof of Dehn–Hadwiger in [DO] is technically correct. However, constructing a d-function $f : \mathbb{R} \to \mathbb{Q}$ (whose domain is $\mathbb{R}$ as an infinite-dimensional $\mathbb{Q}$-vector space) is difficult, rendering the result almost useless for our purposes. (This also affects the proof of Corollary 1.62 and the exposition in Example 1.64.) As such, I gave an alternate approach following [AZ]. My notes are available on the course website.

In the definition of dihedral angle on page 26 of [DO], a minus sign is missing from the equation $n_1 \cdot n_2 = -\cos\theta$.