Math 4990 Problem Set 3

Due Tuesday, Sep 23, 2014 in class

Please refer to previous problem sets for instructions, including but not limited to the collaboration policy.

Assignment

Liberally peruse **pages 22–27, 30–32** of [DO].

You may use Dehn–Hadwiger theorem, which we will prove next lecture, and Sydler theorem, which we will not prove.

[DO] Exercises 1.47, 1.48, 1.51, and 1.65.

Problem 1. Two polygons are **∂-congruent** if one can be decomposed into polygonal pieces and rearranged to form the other, such that the boundary points remain on the boundary. Prove that polygons of the same area and perimeter are ∂-congruent.

Problem 2. A polygon is **centrally symmetric** if it is invariant under a rotation by π (or equivalently, reflection through a point). Two polygons are **T-congruent** if one can be decomposed into polygonal pieces and rearranged *by translations alone* to form the other; *i.e.*, rotations are not allowed.

Prove or disprove: Centrally symmetric polygons with equal area are T-congruent.

Problem 3. Take an n-gon, a translated copy of it (not in the same plane), and join corresponding vertices by edges to form n new faces (necessarily parallelograms). This polyhedron with $n + 2$ faces is called a **prism**. Prove that prisms are scissors congruent to cubes (of the same volume). Do not assume that the prisms are *right* prisms.