Math 1271-040 Midterm Exam 3

Name: _____

ID: _____

TA: _____

Section: _____

1. **Do not open the exam until instructed.**
2. There are 5 problems, each on a single page. Make sure no pages are missing.
3. You have 50 minutes.
4. Each problem is worth 6 points, equally distributed among its parts. As the problems are of varying difficulty level, if you are stuck, you may wish to skip ahead and do other parts first.
5. Organize your work clearly and show an appropriate amount of detail. Illegible scribbles or unsubstantiated correct answers will receive little or no credit.
6. You may (but do not need to) use a scientific calculator.
7. No books, notes, graphing calculators, mobile phones, computers, Rubik's cubes, or other devices allowed.

8. **Arithmetic expressions of numbers need not be simplified:**
   e.g., $1^{1/2} + 2^1 - 3^{-1} + \sqrt{4} + 3\pi - 2\pi + e^3 e^5$ is fine.

Problem 1 (6 points)	
Problem 2 (6 points)	
Problem 3 (6 points)	
Problem 4 (6 points)	
Problem 5 (6 points)	
$\sum$ (30 points total)	

Problem 1. Evaluate the following indefinite integrals.

(a)

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

(b)

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx$$

Problem 2. A particle moves along a line so that its velocity $v(t)$ at time t is given by

$$v(t) = \frac{3t - 6}{\sqrt{t}}.$$

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

(b) Find the distance travelled during the same time period $1 \leq t \leq 4$.

Problem 3. Find the area of the region enclosed by the curves $x = y^4$, $y = \sqrt{2 - x}$, and $y = 0$. [**Hint:** Sketch the region.]

Problem 4. Evaluate the following expressions involving the definite integral.

(a)

$$\int_0^1 xe^{-x^2}\,dx$$

(b)

$$\int_{-23}^{23} \left(1 + x^2 \sin x + x^4 \sin x + x^6 \sin x\right)\,dx$$

(c)

$$\frac{d}{dx}\int_{1-2x}^{x^2-1} \sin\sqrt{t}\,dt$$

Problem 5.

(a) Suppose $f(x)$ is continuous on the interval $[a, b]$. Write down a definition of the definite integral

$$\int_a^b f(x)\,dx$$

using a limit of Riemann sums. You may use right endpoints.

(b) Use the definition above to evaluate the integral

$$\int_1^3 (x-5)\;dx.$$

Do not use FTC2 or any other method. These summation formulae may be helpful:

$$\sum_{i=1}^{n} 1 = n, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$