

**Math 1271-040 Midterm Exam 1****Name:** \_\_\_\_\_**ID:** \_\_\_\_\_**TA:** \_\_\_\_\_**Section:** \_\_\_\_\_

1. **Do not open the exam until instructed.**
2. There are 5 problems, each on a single page. Make sure no pages are missing.
3. You have 50 minutes.
4. Each problem is worth 6 points, equally distributed among its parts. As the problems are of varying difficulty level, if you are stuck, you may wish to skip ahead and do other parts first.
5. Organize your work clearly and show an appropriate amount of detail. Illegible scribbles or unsubstantiated correct answers will receive little or no credit.
6. You may (but do not need to) use a scientific calculator.
7. No books, notes, graphing calculators, mobile phones, computers, Rubik's cubes, or other devices allowed.

Problem 1 (6 points)	
Problem 2 (6 points)	
Problem 3 (6 points)	
Problem 4 (6 points)	
Problem 5 (6 points)	
$\sum$ (30 points total)	

**Problem 1.** Let  $f(x) = x^3 + 3x^2 + 5$ .

(a) Calculate  $f''(x)$ .

(b) Suppose  $c$  is a number such that  $f''(c) = 0$ . Determine the value of  $c$ .

(c) Find an equation of the tangent line to the graph of  $f(x)$  at  $x = c$ , where  $c$  is the value determined in part (b).

**Problem 2.** Evaluate the limits. Simplify answers but leave them exact (e.g., do not use decimal approximations). Answers could be  $\infty$ ,  $-\infty$ , or “does not exist.”

(a)  $\lim_{\theta \rightarrow \pi} \theta^2 + \cos \theta$

(b)  $\lim_{x \rightarrow 2} \frac{e^{5x} - e^{10}}{x - 2}$

(c)  $\lim_{x \rightarrow -\infty} x + \sqrt{x^2 - 3x}$

**Problem 3.** Differentiate. It is not necessary to simplify answers.

(a)  $f(x) = (5x - 7)^2(2x^{23} - x)^3$

(b)  $f(x) = e^{\sqrt{x-e^{3x}}}$

(c)  $f(x) = \frac{3x^8 + 2x - 7}{\sqrt[3]{x}}$

**Problem 4.**

(a) Write down the definition of the derivative of a function  $f$  at a point  $a$ .

(b) Find the derivative of  $f(x) = \frac{1}{\sqrt{5x}}$  using the definition of the derivative. Do not use differentiation rules.

**Problem 5.** Prove the following statements. Justify answers and cite theorems used.

(a)  $\lim_{t \rightarrow 0} t^3(t + \cos \frac{1}{t^2}) = 0$

(b) The function  $f(x) = \sin x + \frac{\pi}{x}$  has a root in the interval  $(-10\pi, 10\pi)$ .