100 Points Total.

Problems 1-4: Multiple Choice. Select the ONE correct answer. No work needs to be shown on this section. 5 pts each.

1. Use Newton's Method to approximate a real root of $f(x) = x^3 + x - \frac{1}{x}$. Compute the second approximation x_2 using the first approximation $x_1 = 1$.

   A. $x_2 = -4$

   B. $x_2 = \frac{4}{5}$

   C. $x_2 = \frac{6}{5}$

   D. $x_2 = 6$

2. Evaluate $f(x) = \int_{-\pi/4}^{\pi/4} \cos^3(x) \cdot \sin^3(x)\, dx$.

A. $-\frac{1}{24}$

B. 0

C. $\frac{1}{24}$

D. $\frac{1}{12}$

3. Let $h(x) = \int_1^{\sqrt{x}} \frac{t^2}{t^4+1}\, dt$. Find $h'(x)$.

A. $h'(x) = x \cdot \arctan\left(\sqrt{x}\right)$

B. $h'(x) = \frac{x}{x^2+1}$

C. $h'(x) = \frac{x^2}{x^4+1}$

D. $h'(x) = \frac{x}{2\sqrt{x}(x^2+1)}$

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4. What is the average value of the function $f(x) = 4x^3 - 6x^2 - 1$ on the interval $[0,3]$?

A. 8

B. 9

C. 24

D. 26

For the remaining problems, show all steps. Unsupported answers will receive little to no credit.

5. (8 pts) Find $f(x)$ if we are told that $f''(x) = 6x^2 + 6x - 8$, along with initial conditions $f(0) = 3$ and $f(1) = 2$.

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6. (10 pts) Using the <u>limit definition of an integral</u>, evaluate the following:

$$\int_0^2 (5x - 4)dx$$

Use right endpoints (the standard form). You will **not** receive credit for using the Fundamental Theorem of Calculus or a geometric argument. The following formulas may be used if needed:

$$\sum_{i=1}^{n} 1 = n, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

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7. (6 pts) Suppose $f(x)$ is a continuous function such that $\int_0^5 f(x)dx = 8$, $\int_0^7 f(x)dx = 10$, and $\int_2^7 f(x)dx = 3$.

What is the value of $\int_2^5 f(x)dx$?

8. (6 pts) Evaluate the following integral. You may use the Fundamental Theorem of Calculus, u-substitution, symmetry, and/or geometry to justify your answer.

$$\int \frac{\left(\sqrt{x} - 3\right)^2}{x} dx$$

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9. (6 pts each) Evaluate the following integrals. You may use the Fundamental Theorem of Calculus, u-substitution, symmetry, and/or geometry to justify your answers.

a. $\displaystyle\int x^2 e^{2x^3+1} dx$

b. $\displaystyle\int_{-4}^{4} \left(x + \sqrt{16-x^2} \right) dx$

c. $\displaystyle\int_{0}^{\pi/2} \frac{\cos(x)}{1 + \sin^2(x)} dx$

10. Let the velocity of a particle moving in a straight line at time t be given by the function $v(t) = -2t + 4$.

a. (5 pts) Find the net displacement of the particle during the time period from $t = 0$ to $t = 3$.

b. (7 pts) Find the total distance traveled by the particle during the time period from $t = 0$ to $t = 3$.

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11. (10 pts) Find the area of a region between the curves $y = 2x$ and $y = x^2$. Draw a sketch of the area in question.

12. (10 pts) Find the volume of the solid created by rotating the region between the curves $y = 2x$ and $y = x^2$ about the x-axis. Depending on which method you use, include a sketch of the region and a typical "disk/washer" or "cylindrical shell."