Math 1271 Exam 2, Spring 2015 Name _____

100 Points Total.

Problems 1-4: Multiple choice. Select the ONE correct answer. No work needs to be shown on this section. 5 pts each.

1. Let $F(x) = [g(x)]^2 + x \cdot g(x) - 6$. Suppose $g(2) = 5$ and $g'(2) = -1$. Find $F'(2)$.

A. $F'(2) = 15$

B. $F'(2) = -7$

C. $F'(2) = 13$

D. $F'(2) = 9$

2. Let $f(x) = e^{1-\cos(\ln(3x))}$. What is $f'(x)$?

A. $f'(x) = e^{1-\cos(\ln(3x))}$

B. $f'(x) = \frac{1}{x}\sin(\ln(3x))\, e^{1-\cos(\ln(3x))}$

C. $f'(x) = \frac{1}{3x}\sin(\ln(3x))\, e^{1-\cos(\ln(3x))}$

D. $f'(x) = \frac{1}{3x}\sin(x)\, e^{x}$

3. Find $\lim_{x\to 0} \frac{e^{3x} - 1 - 3x}{2x^2}$

A. 0

B. ∞

C. $-\frac{3}{2}$

D. $\frac{9}{4}$

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4. What is the absolute maximum of the function $f(x) = 3x^{2/3} - 4x$ on the interval $[0,1]$?

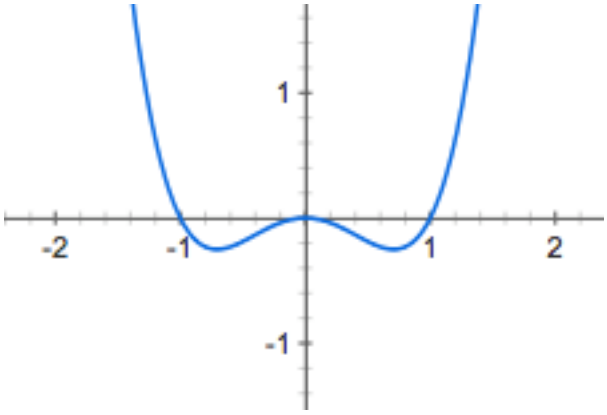
A. 0

B. 0.125

C. 0.25

D. 1

5. The following is the graph of the $f'(x)$, the **derivative** of $f(x)$. Use the graph below to answer the following questions about $\boldsymbol{f(x)}$ **(<u>the original function</u>)**.



a. (4 pts) Over which intervals is $f(x)$ increasing and where is it decreasing?

b. (3 pts) For which value(s) of x does $f(x)$ have a local maximum or minimum?

c. (3 pts) Is the graph of $f(x)$ concave upward or concave downward at $x = -1$? Explain.

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For the remaining problems, show all steps. Unsupported answers will receive little to no credit.

6. (4 pts) Let $f(x) = \arctan\left((2x-3)^{3/2}\right)$. Find $f'(x)$.

7. Suppose $1 + \sin(xy) = x^2 - y$

a. (6 pts) Find y' using implicit differentiation.

b. (4 pts) Use your result from part a to find the equation of the line tangent to the curve at the point $(1,0)$.

8. (10 pts) A balloon is rising straight up at a constant speed of 5 ft/sec. A boy rides a bicycle along a straight road at a speed of 15 ft/sec. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 seconds later?

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9. Let $f(x) = x^{1/x}$.

a. (6 pts) Find $f'(x)$. Your final answer should be a function of x alone.

b. (5 pts) Find $\lim_{x\to\infty} f(x)$.

10. (10 pts) A box whose length is 3 times the width must have a volume of 12 cm^3. The cost of material for the sides is \$6/cm^2 and the top and bottom costs \$2/cm^2. Find the dimensions that will minimize the cost of the box.

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11. (6 pts) Suppose $f(x)$ is a continuous function on the interval $[1,5]$. Suppose also that $f(1) = 2$, and that $-1 \leq f'(x) \leq 3$ for all values of x in the interval. Using the Mean Value Theorem, find the maximum and minimum possible values of $f(5)$.

12. (5 pts) Let $f(x) = \sqrt{x+1} + \sin\left(\frac{\pi}{2}x\right)$. Use a linear approximation at $x = 3$ to estimate $f(3.2)$.

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13. Let $f(x) = \frac{x^4 - 4x^3}{3}$

a. (2 pts) Find the x-intercept(s) of $f(x)$.

b. (4 pts) Find the intervals of increase/decrease. Locate any local max/min.

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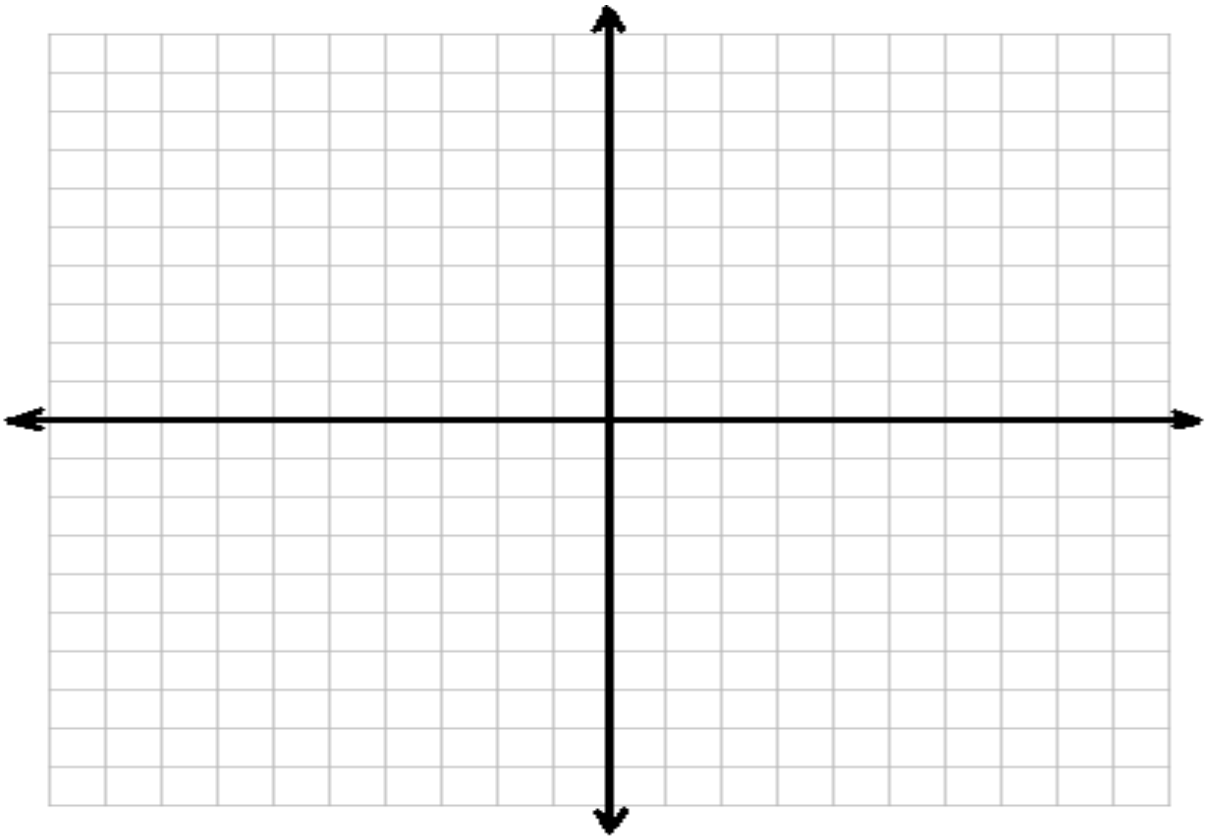
(Continued from previous page) Let $f(x) = \frac{x^4 - 4x^3}{3}$

c. (4 pts) Find the intervals of concavity and locate any inflection points.

d. (4 pts) Sketch the graph of f(x). **Label** the intercepts, any local max/min values and inflection points.



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