Math 1271 Exam 1, Spring 2015 Name _____

100 Points Total.

Problems 1-4: Multiple choice. Select the ONE correct answer. No work needs to be shown on this section. 5 pts each.

1. Which of the following statements is true?

A. If a function is continuous at a, then it is differentiable at a.

B. If a function is differentiable at a, then it is continuous at a.

C. If $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$, then $f(x)$ is continuous at a.

D. None of the above statements are true.

2. Find the **vertical** asymptote(s) of the rational function, $f(x) = \frac{x^2+5x-24}{2x^2-18}$.

A. $y = \frac{1}{2}$

B. $x = -3$

C. $x = 3$

D. $x = -3$ and $x = 3$

3. $f(x) = \frac{\cos x}{1-\sin x}$. What is $f'(x)$?

A. $f'(x) = \frac{1}{1-\sin x}$

B. $f'(x) = \frac{1}{(1-\sin x)^2}$

C. $f'(x) = -\frac{1}{1-\sin x}$

D. $f'(x) = \frac{\sin^2 x - \cos^2 x}{(1-\sin x)^2}$

____ / 15

4. Find the **horizontal** asymptote(s) of the function $f(x) = \frac{\sqrt{x^2+1}}{x-2}$.

A. $x = 2$

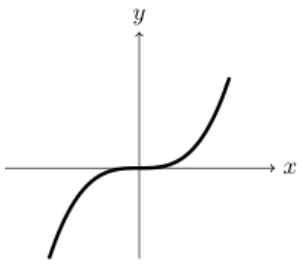
B. $y = -1$

C. $y = 1$

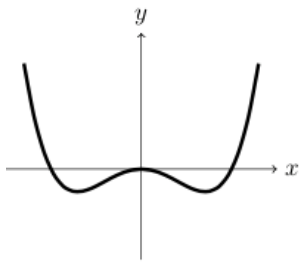
D. $y = -1$ and $y = 1$

5. Match the following graphs (i-iv) to the graphs of their derivatives (A-D). 8 pts total.

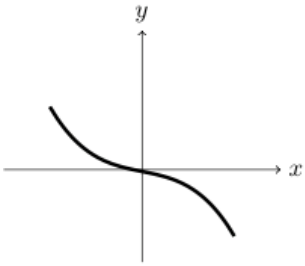
i.



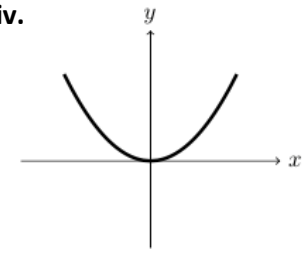
ii.



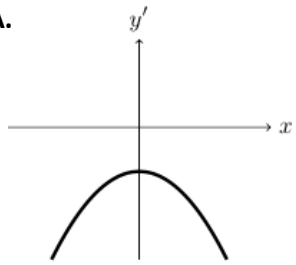
iii.



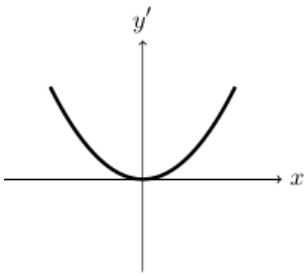
iv.



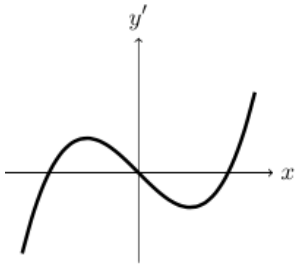
A.



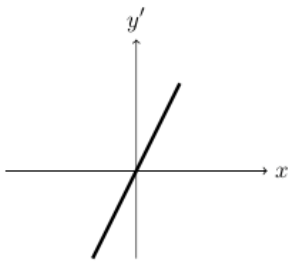
B.



C.



D.



Graph i : _____ **Graph ii : _____** **Graph iii : _____** **Graph iv : _____**

____ /13

For the remaining problems, show all steps. Unsupported answers will receive little to no credit.

6. (7 pts) Let $f(x) = \sqrt{3x + 5}$. Using the definition of a derivative (difference quotient), find $f'(x)$.

7. (6 pts) Show that $f(x) = x^2 + \sin x - 3$ has at least one real root. You must justify your answer, but it is not necessary to actually calculate any roots.

8. Suppose a ball is thrown upward from a 150ft ledge with an initial velocity of 128 ft/sec. The height of the ball (in ft) after t seconds is given by the function $h(t) = -16t^2 + 128t + 150$.

a. (3 pts) What is the velocity of the ball after 2 seconds?

b. (3 pts) At what time is the velocity of the ball 0?

c. (3 pts) What is the height of the ball when the velocity is 0?

d. (3 pts) What is the acceleration of the ball after 2 seconds? After 3 seconds? After 5 seconds?

9. (6 pts) Use the Squeeze Theorem to evaluate the following limit. Recall that $\cos x$ is always between -1 and 1.

$$\lim_{x \to \infty} \frac{\cos x}{\sqrt{x}}$$

10. Evaluate the following limits. "∞", " $-\infty$", and DNE are all distinct answers.

a. (4 pts) $\lim_{x\to 4} \dfrac{x^2-6x+8}{x^2+3x-28}$

b. (4 pts) $\lim_{x\to -3^+} \dfrac{4x-1}{x+3}$

c. (4 pts) $\lim_{x\to \infty} \dfrac{x}{\sqrt{4x^2+x}+2x}$

____ / 12

11. Suppose f and g are functions such that $f(1) = -5$, $g(1) = 3$, $f'(1) = 2$, and $g'(1) = -4$.

a. (4 pts) Find $h'(1)$ if $h(x) = f(x) \cdot g(x)$.

b. (4 pts) Find $F'(1)$ if $F(x) = \dfrac{f(x)}{g(x)}$

12. Differentiate each of the following functions.

a. (3 pts) $f(x) = x^{15} - 7x^3 + e^2 + \tan x$

b. (3 pts) $g(x) = e^x(5x - 8)$

c. (3 pts) $h(x) = \dfrac{x^3 - 8x + \sqrt{x}}{x^2}$

13. (6 pts) Find the point on the curve $f(x) = 3x^2 - 5x + 8$ where the tangent line is parallel to the line $y = 7x - 9$.

14. (6 pts) Let

$$f(x) = \begin{cases} e^x(x + 6\cos x) & x < 0 \\ a(x - 2) & x \geq 0 \end{cases}$$

Find all values of a such that $f(x)$ is continuous everywhere.