MATH 32A DISCUSSION

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1. Parametric Curves and Vector Functions

1.1. **Exercise 11.1.14.** Given parametric equation $x = e^t - 1$, $y = e^{2t}$, find the Cartesian equation.

Solution. Obviously $y = (x+1)^2$. Notice that $e^t > 0$, so the graph is only the portion $x > -1$, $y > 0$. □

1.2. **Exercise 11.1.41.** Let A and B be circles of radii a and b, respectively, centred at the origin. Let a ray from the origin intersect the circles and draw perpendicular as in the figure (in the textbook). Find the collection of such points P.

Solution. It is obvious that $x = a\cos\theta$ and $y = b\sin\theta$. So, eliminating θ, we have $(x/a)^2 + (y/b)^2 = 1$ is an ellipse. □

1.3. **Exercise 14.1.1.** Find the domain of the vector function

$$\mathbf{r}(t) = \left\langle \sqrt{4-t^2}, e^{-3t}, \ln(t+1) \right\rangle.$$

Solution. We need $4 - t^2 \geq 0$ and $t + 1 > 0$, so we get $t \in [-2, 2] \cap (-1, \infty) = (-1, 2]$. □

1.4. **Exercise 14.1.41.** Suppose the trajectories of two particles are given by the vector functions $\mathbf{r}_1(t) = \left\langle t^2, 7t-12, t^2 \right\rangle$ and $\mathbf{r}_2(t) = \left\langle 4t-3, t^2, 5t-6 \right\rangle$, respectively. Do the particles collide?

Solution. If they do, then $\mathbf{r}_1(t) = \mathbf{r}_2(t)$ for some t. Directly solving we get $4t - 3 = t^2 = 5t - 6$ so $t = 3$ is possible. Checking, we get indeed $\mathbf{r}_1(3) = \mathbf{r}_2(3)$, so they do collide. □

2. Equations of Lines and Planes

2.1. **Basics.** If a line L passes through the point $P_0(x_0, y_0, z_0)$, represented by the vector $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$ and has direction $\mathbf{v} = \langle a, b, c \rangle$, which are the *direction numbers*, then the *vector equation* of L is $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$, where $\mathbf{r} = \langle x, y, z \rangle$. The *parametric equations* are just the three components $x = x_0 + at$, $y = y_0 + at$, $z = z_0 + at$. The *symmetric equations* are $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$.

A plane that passes through P_0 and has normal vector $\mathbf{n} = \langle a, b, c \rangle$ has *vector equation* $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$, and has the *scalar equation* $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$. The *linear equation* is $ax + by + cz + d = 0$, where $d = -(ax_0 + by_0 + cz_0)$ can be found by collecting terms.

The distance between $P_1(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

2.2. **Exercise 13.5.53.** Determine if the planes given by $x = 4y - 2z$, $8y = 1 + 2x + 4z$ are parallel, perpendicular, or neither. If neither, find the angle between them.

Solution. Normal to the planes are $\mathbf{n}_1 = \langle -1, 4, -2\rangle$ and $\mathbf{n}_2 = \langle 2, -8, 4\rangle$, respectively. To find the angle θ between the planes, we find the angle θ between the normals by using the dot product: $\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1|\,|\mathbf{n}_2| \cos\theta$. Notice $-2\mathbf{n}_1 = \mathbf{n}_2$ so they are parallel. Or we could calculate and get $\theta = 0$. □

2.3. **Exercise 13.5.73.** Show that the distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is $D = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$.

2.4. **Exercise 13.5.75.** Show that the lines with symmetric equations $x = y = z$ and $x + 1 = y/2 = z/3$ are skew, and find the distance between these lines.

2.5. **Exercise 13.5.77.** If a, b, and c are not all 0, show that the equation $ax + by + cz + d = 0$ represents a plane and $\langle a, b, c\rangle$ is a normal vector to the plane.