MATH 32A DISCUSSION

JED YANG

1. Introduction

Lecture 1

- Instructor: Steve Butler.
- Location: BUNCHE 1209B.

Sections 3C and 3D

- Email: `mailto:jedyang@ucla.edu`.
- Office: MS 6617A.
- Office hours: TR 13:00–13:30.
- Discussion Location: MS 6229.
- Website: `http://www.math.ucla.edu/~jedyang/32a.3.10s/`.
- SMC: Mar. 31–Jun. 3, M–R 10:00–15:00, MS 3974, M 14:00–15:00.

2. Administration

- HW due Fridays in lecture, can turn in early to me, and I will hand back in section.
- Textbook: Stewart, Multivariable Calculus, 6e, 2008.
- Confirm office hour.

3. 3-D Coordinates

3.1. **Exercise 13.1.8.** Find the lengths of the sides of the triangle PQR, where $P(2,-1,0)$, $Q(4,1,1)$, $R(4,-5,4)$. Is it a right triangle? Is it an isoceles triangle?

Solution. $|PQ|^2 = 2^2+2^2+1^2 = 9$, $|QR|^2 = 0^2+6^2+3^2 = 45$, $|PR|^2 = 2^2+4^2+4^2 = 36$. Obviously not isoceles, but is right triangle since $9+36 = 45$. □

3.2. **Exercise 13.1.11.** Find an equation of the sphere with centre $(1,-4,3)$ and radius 5. What is the intersection of this sphere with the xz-plane?

Solution. The equation is $(x-1)^2+(y+4)^2+(z-3)^2 = 5^2$. If we set $y = 0$, we get $(x-1)^2+(z-3)^2 = 3^2$, a circle in the xz-plane. □

3.3. **Exercise 13.1.14.** Find an equation of the sphere that passes through the point $(4,3,-1)$ and has centre $(3,8,1)$.

Solution. We can think of $(x-3)^2+(y-8)^2+(z-1)^2 = (4-3)^2+(3-8)^2+(-1-1)^2$ as the equation of the sphere on the left and calculating the radius on the right, or plugging the point it ought to pass through to the same equation. □

3.4. **Exercise 13.1.28.** Describe in words the region of $\mathbb{R}^3$ represented by the equation $z^2 = 1$.

Solution. Notice $z^2 = 1$ means $z = \pm 1$, so we have two planes parallel to the xy-plane. $\square$

3.5. **Exercise 13.1.38.** Consider the points P such that the distance from P to $A(-1, 5, 3)$ is twice the distance from P to $B(6, 2, -2)$. Show that the set of all such points is a sphere, and find its centre and radius.

Solution. We have $|PA| = 2\,|PB|$, squaring, we get $|PA|^2 = 4\,|PB|^2$. So we want to simplify $(x+1)^2+(y-5)^2+(z-3)^2 = 4[(x-6)^2+(y-2)^2+(z+2)^2]$. Expanding and collecting terms, we get $3x^2-50x+3y^2-6y+3z^2+22z+141 = 0$. Dividing by 3 then completing the squares, we get $(x-25/3)^2+(y-1)^2+(z+11/3)^2 = 332/9$, a circle of radius $\frac{2}{3}\sqrt{83}$. $\square$

4. Vectors

4.1. **Exercise 13.2.18.** Given $\mathbf{a} = 4\mathbf{i}+\mathbf{j}$, $\mathbf{b} = \mathbf{i}-2\mathbf{j}$. Find $\mathbf{a}+\mathbf{b}$, $2\mathbf{a}+3\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a}-\mathbf{b}|$.

Solution. $\mathbf{a}+\mathbf{b} = 5\mathbf{i}-\mathbf{j}$, $2\mathbf{a}+3\mathbf{b} = 11\mathbf{i}-4\mathbf{j}$, $|\mathbf{a}| = \sqrt{4^2+1^2}$, $\mathbf{a}-\mathbf{b} = 3\mathbf{i}+3\mathbf{j}$, so $|\mathbf{a}-\mathbf{b}| = 3\sqrt{2}$. $\square$

4.2. **Exercise 13.2.35.** Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.

Solution. Tangent line has slope $y' = 2x$ with $x = 2$, so slope 4. Take $\langle 1, 4\rangle$ and normalise to get $\langle 1/\sqrt{17}, 4/\sqrt{17}\rangle$. We also get the negative of that. $\square$

4.3. **Exercise 13.2.45.** Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

Solution. Let $\overrightarrow{AB} = 2\mathbf{a}$, $\overrightarrow{BC} = 2\mathbf{b}$. Then the vector representing the midline is $\mathbf{a}+\mathbf{b}$ whereas the third side is $\overrightarrow{AC} = \overrightarrow{AB}+\overrightarrow{BC} = 2(\mathbf{a}+\mathbf{b})$. $\square$