Math131A Set 3

Due on Monday, July 15, 2013, before the Midterm Exam.

Collaboration is encouraged, as long as you write your own solutions and write down the name of your collaborators.

11. Subsequences

11.1. Consider the sequences defined as follows: $a_n = 1/n^2$, $b_n = (-1)n^3$, $c_n = \frac{3n+2}{4n-7}$, $d_n = 23^{(-1)^n}$.

- (a) For each sequence, give an example of a monotone subsequence.
- (b) For each sequence, give its set of subsequential limits.
- (c) For each sequence, give its lim sup and lim inf.
- (d) Which of the sequences converges? diverges to $+\infty$? diverges to $-\infty$?
- (e) Which of the sequences is bounded?

11.2. Suppose (a_n) is a subsequence of (b_n) , and (b_n) is a subsequence of (c_n) . Prove that (a_n) is a subsequence of (c_n) .

11.3. Prove that $\liminf s_n = -\limsup(-s_n)$.

12. More subsequences

Do exercises 12.1, 12.4, 12.8, 12.10 in Ross.

14. Series

Do exercises 14.6, 14.8, 14.9 in Ross.

14.1. Determine which of the following series converge. Justify your answers.

(a)
$$\sum \cos n$$

(b) $\sum_{n=10}^{\infty} (\log n)^{-2}$
(c) $\sum (\sqrt{n+1} - \sqrt{n})$
(d) $\sum \frac{n^3}{1.7^n}$

15. More series

Do exercises 15.6, 15.7 and 15.8 in Ross.