Homework 40: multivariable optimization

Print and attach your *Mathematica* work; label each part with the associated problem and part numbers to make it easier for the grader to find your work.

(1) Find all second-order partial derivatives for the following functions by hand. (Of course, you may check your work on *Mathematica* if you wish.)
  (a) $f(x, y) = 2(xy + y)^3$
  (b) $f(x, y) = \ln(xy)$

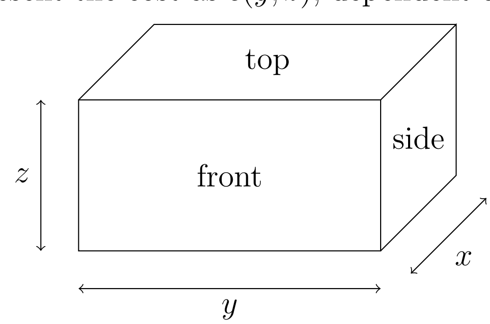
(2) Consider the function $f(x, y) = 8y^2 + 4x^4 - 16xy$.
  (a) Find all critical points by hand, then check by finding all critical points using *Mathematica*.
  (b) Plot with `Plot3D` and `ContourPlot` on *Mathematica*.
  (c) Use your plots to classify the critical points as local maximum, local minimum, or saddle points. Explain in a short sentence. (You may need to make additional contour plots close to the critical points.)

(3) For this problem, you may do all your work on *Mathematica*. Find all critical points and then use the second derivative test to classify the critical points for

$$f(x, y) = x^5 + y^4 - 5x - 32y - 3.$$

Plot the function on *Mathematica* to see if your algebraic analysis agrees with the graphical representation of the function.

(4) A rectangular box of volume 24 cubic feet is to be constructed. Materials for the sides costs 50 cents per square foot, for the front and back costs 75 cents per square foot, and for the top and bottom costs 1 dollar per square foot. What dimensions should the box have to minimize its cost?
  (a) Set up the function by hand. First, use variables x, y, and z to represent the total cost of construction. Then, use another equation involving these variables in order to represent the cost as $c(y, z)$, dependent on y and z only (and not x).



  (b) Use any method you like (*Mathematica* or by hand) to find the critical points. Then use the second derivative test to classify the critical points.