CS 252

W, 8 May 2024

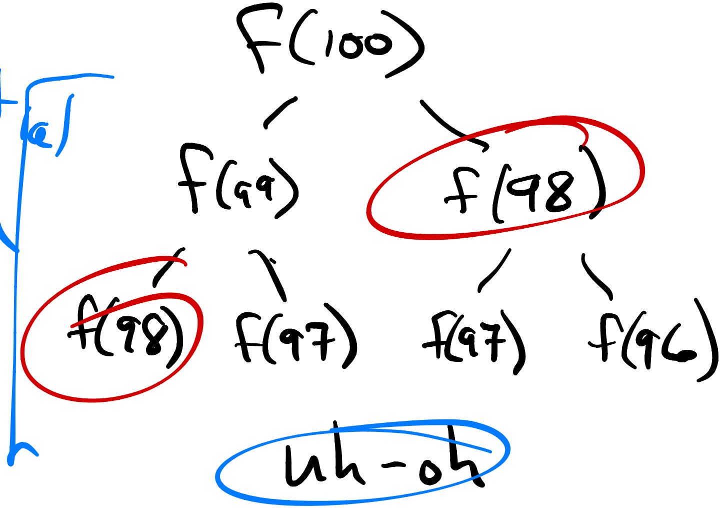
# Fibonacci #s

$f(0) = 1$

$f(1) = 1$

recursive $f(n) = f(n-1) + f(n-2)$ for $n > 1$

exponential growth




| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ... |
|---|---|---|---|---|---|---|---|---|---|
| $f(n)$ | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | ... |

Solution: compute small values first & gradually build up big ones

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$M(5)$ = max revenue for a piece of wire of length 5

12 (3" + 1" + 1"
↳ 8 + 2 + 2 $) (5" → 12 $)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

What are we optimizing?

Total revenue from length n of wire

Allow integer cuts

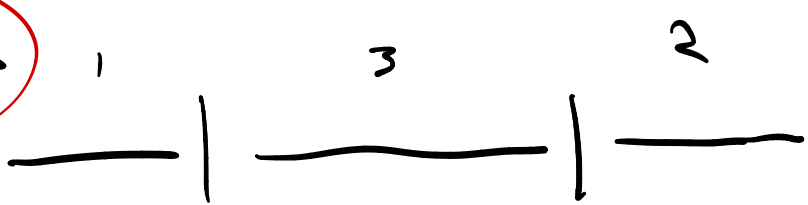
Maximize: $\sum_{i=1}^{k} p[\text{length of piece } i]$

$m=6$ We're never dealing lengths $> m$

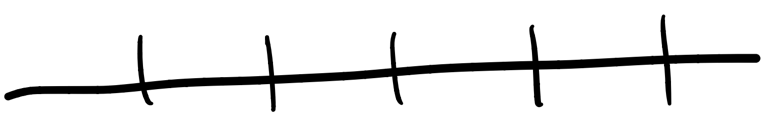
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

Search space? Set of combinations of cuts.

$\boxed{n=6}$




← 1 element of the search space



$n-1$, 5 places I could make cuts

# combinations of cuts? $2^5 = 2^{n-1}$

Problem: maximize \$

$$\sum_i p[\text{piece } i]$$

Search space

sets of cuts, $2^{n-1}$ possibilities

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P$ | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

Let $M(n)$ = the maximum money obtainable by cutting up a wire of length $n$.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\ldots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$

Best you can get from 1st cut $i$, then cut up the rest

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\dots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$

$$M(1) = \max\left(p[1] + M(0)\right) = \max(2+0)$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| M | 0 | 2 | | | | | |
| P | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\dots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| M | 0 | 2 | 4 | | | | |
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(2) = \max\left( p[1] + M(1),\ p[2] + M(0) \right)$$

$$= \max(2+2,\ 3+0)$$

$$= 4$$

$$M(k) = \max_{i=1,\dots,k} \left( p[i] + M(k-i) \right)$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\dots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$

$$M(3) = \max\left( p[1]+M(2),\ p[2]+M(1),\ p[3]+M(0) \right)$$

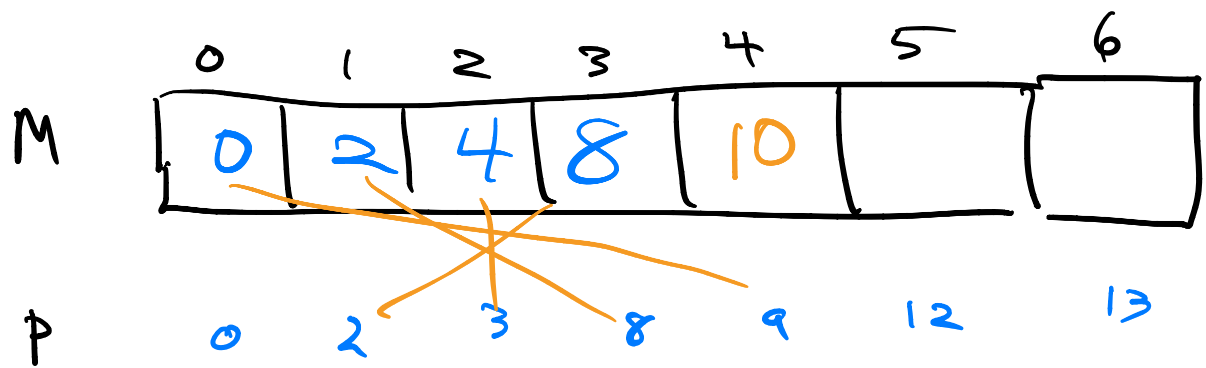
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| M | 0 | 2 | 4 | 8 | | | |
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$= \max\left( 2+4,\ 3+2,\ 8+0 \right)$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\dots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$



| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\dots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| M | 0 | 2 | 4 | 8 | 10 | 12 | |
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\ldots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| M | 0 | 2 | 4 | 8 | 10 | 12 | 16 |
| P | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = O(n^2)$$

$M(1) \sim 1$ const time

$M(2) \sim 2$ " "

$M(3) \sim 3$

$$M(n) \approx 1+2+3+\dots+n = \frac{n(n+1)}{2} = O(n^2)$$