Slithering the Link

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March 12, 2016
Roadmap

What is a Slitherlink Puzzle?

How to Define a Slitherlink Puzzle

How to Solve a Slitherlink Puzzle

How to Make a Slitherlink Puzzle
What is Slitherlink?

Logic puzzle developed by Nikoli
Played on:
  ▶ a rectangular lattice of dots, creating "cells"
  ▶ with some cells containing numbers
What is Slitherlink?

Objective of the game is to create a single loop throughout the puzzle where:

- the final solution is a continuous line that does not cross itself
- each numbered cell corresponds to the number of solution lines around it
- the puzzle should have ONLY ONE unique solution
Solving a Slitherlink Puzzle
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Solving a Slitherlink Puzzle

Conceptis Puzzles Slitherlink Techniques
Solving a Slitherlink Puzzle
Solving a Slitherlink puzzle is an NP-complete problem, as well as determining if there are multiple solutions.

On the NP-completeness of the Slither Link Puzzle Takayuki YATO
Puzzle Representation

Components of a puzzle:
- the grid
  - lines
  - numbers
- rules and contradictions
- contours
- what it means to be solved
The Grid

M by N grid has 3 2D arrays:
- M by N 2D array for numbers; values 0 to 3 or empty
- M + 1 by N 2D array for horizontal lines; values line, x, or empty
- M by N + 1 2D array for vertical lines; values line, x, or empty

A rule-based approach to the puzzle of Slither Link. Stefan Herting.
Rules and Contradictions

Each rule has:
▶ dimensions
▶ prerequisites
▶ consequences

Each contradiction has:
▶ dimensions
▶ prerequisites
Examples of Rules
Static Rules

Static rules are rules that do not contain lines or x’s in their prerequisites. We identified 3 static rules.
We chose to cover rules that are at most 3 by 3 in dimension, and contradictions that are at most 2 by 2 in dimension.
Contours

- Use 2D array to keep track of contour endpoints.
- Update endpoints as we add lines.
- Keep track of the number of open and closed contours as we add lines.

Table 1: Contour Endpoint Array

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3,1</td>
<td></td>
<td>0,1</td>
</tr>
<tr>
<td>1,2</td>
<td>0,2</td>
<td>1,3</td>
</tr>
<tr>
<td>2,2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{numClosed} = 0 \\
\text{numOpen} = 3
\]
How Can We Tell Our Grid is Solved?

- Every number in the grid is satisfied
- There is exactly one closed loop, and no open loops.
How Can We Tell Our Grid is Solved?

- Every number in the grid is satisfied.
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Applying Rules

- for every position in the grid...
  - for every defined rule...
    - for every orientation...

Do the prerequisites in the rule match where we’re looking at on the grid?
- If so, add consequences to the grid.
Guessing

Method:

1. Find a particular open position
2. Guess that position is a Line
   2.1 Run deterministic rules
   2.2 If there's a contradiction, we know the position is an X
3. Guess that position is an X
   3.1 Run deterministic rules
   3.2 If there's a contradiction, we know the position is a Line
4. If neither results in a contradiction, take the intersection of the two resulting grids
Guessing

Method:

1. Find a particular open position
Guessing

Method:

1. Find a particular open position
2. Guess that position is a **LINE**
Guessing

Method:

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Guessing

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Recursive Guessing

If single guesses don’t result in anything, we can nest our guesses. New information from nested guesses propagates out to the canonical grid.
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We call $n$ nested guesses a ”depth $n$ guess”.
Example of a depth 2 guess

\textsc{Grid}
Example of a depth 2 guess

GRID
  |
  A
Example of a depth 2 guess

```
  GRID  
    |    
   A    
  LINE  
       X
```
Example of a depth 2 guess
Example of a depth 2 guess
Example of a depth 2 guess
Example of a depth 2 guess

```
GRID
  |
  A
/  \\nLINE X
  |
  B
  |
LINE X
  |
contradiction
  |
valid
  |
LINE X
  |
C
```
Example of a depth 2 guess

```
GRID
  A
   /
  LINE
    /
   B
      /
     X
   /
   C
    /
   LINE
      /
     X
valid
```
Example of a depth 2 guess

```
GRID
  /
A
  /
LINE X
  /
B X
  /
valid contradiction
```
Example of a depth 2 guess
Example of a depth 2 guess

```
GRID
  |
  A
  |
LINE   X
  |
  B    contradiction
  |
  |
  X
  |
valid
```
Example of a depth 2 guess

```
GRID
<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
LINE
|   |
|   |
| B |
|   |
| X |
```
Repeated guessing algorithm

- If at any point new information is found, restart algorithm
Repeated guessing algorithm

- If at any point new information is found, restart algorithm
- Run deterministic rules
Repeated guessing algorithm

- If at any point new information is found, restart algorithm
- Run deterministic rules
- Run every possible guess (depth 1 guessing)
Repeated guessing algorithm

- If at any point new information is found, restart algorithm
- Run deterministic rules
- Run every possible guess (depth 1 guessing)
- Run every possible guess, and within each guess, make all possible guesses (depth 2 guessing)
Repeated guessing algorithm

- If at any point new information is found, restart algorithm
- Run deterministic rules
- Run every possible guess (depth 1 guessing)
- Run every possible guess, and within each guess, make all possible guesses (depth 2 guessing)
- ...

...
Solving a Slitherlink Puzzle
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Solving a Slitherlink Puzzle
Checking for Multiple Solutions

For every guess we made to get to the solution:

- Go back to the state of the grid before the guess was made, and solve the corresponding grid with the opposite guess at that same spot:
- If the opposite guess eventually leads to a contradiction, we know that the original guess has to be true (given all previous guesses). Continue to check other guesses.
- If the opposite guess eventually leads to one or more solutions, then we know that this grid has more than one solution.

If the opposite of every guess we had to make leads to a contradiction, then we know that the original solution we found is the only one.
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Checking for Multiple Solutions
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Checking for Multiple Solutions
Time Complexity

Important details:

▶ For each depth, $O(n^m)$ new guesses, each taking $O(n^m)$ time.

▶ Guessing at the max depth is by far the most important factor in runtime.

▶ We can maximize this by never filling anything in at lower depths.
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Time Complexity

Overall runtime $O((mn)^d)$ with $d$ bounded above by $O(mn)$

<table>
<thead>
<tr>
<th>Size</th>
<th>Max Depth</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x3</td>
<td>0</td>
<td>0.001111</td>
</tr>
<tr>
<td>3x3</td>
<td>1</td>
<td>0.033743</td>
</tr>
<tr>
<td>3x3</td>
<td>2</td>
<td>1.19998</td>
</tr>
<tr>
<td>3x3</td>
<td>3</td>
<td>57.2004</td>
</tr>
<tr>
<td>3x3</td>
<td>4</td>
<td>2587.52</td>
</tr>
</tbody>
</table>
## Empirical Results: Typical Puzzles

### Table 3: Solve Times

<table>
<thead>
<tr>
<th>Size</th>
<th>Max Depth</th>
<th>Solve Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10x10</td>
<td>1</td>
<td>0.048542 seconds</td>
</tr>
<tr>
<td>10x10</td>
<td>1</td>
<td>0.385348 seconds</td>
</tr>
<tr>
<td>10x10</td>
<td>2</td>
<td>1.77571 seconds</td>
</tr>
<tr>
<td>10x10</td>
<td>2</td>
<td>3.23716 seconds</td>
</tr>
<tr>
<td>10x10</td>
<td>3</td>
<td>150.824 seconds*</td>
</tr>
<tr>
<td>30x25</td>
<td>1</td>
<td>1.95466 seconds</td>
</tr>
<tr>
<td>30x25</td>
<td>1</td>
<td>2.38471 seconds</td>
</tr>
<tr>
<td>30x25</td>
<td>1</td>
<td>4.4892 seconds</td>
</tr>
<tr>
<td>40x30</td>
<td>1</td>
<td>1.97524 seconds</td>
</tr>
<tr>
<td>40x30</td>
<td>3</td>
<td>66.268 seconds*</td>
</tr>
</tbody>
</table>

*These puzzles were determined to have multiple solutions.

Puzzles taken from [nikoli.com](http://nikoli.com), [kakuro – online.com](http://kakuro-online.com), and [puzzle – loop.com](http://loop.com).
Make a Slitherlink Puzzle

Overview
1. Make a loop
2. Fill grid with numbers
3. Remove some numbers
Making a Loop

Start with an empty $m \times n$ grid, a simple rule, and three lists:

1. available
2. expandable
3. unexpandable
Making a Loop

Start with an empty $m \times n$ grid, a simple rule, and three lists:

1. available
2. expandable
3. unexpandable

Start with every location in the grid in available but none in unexpandable and expandable. Then, add one random location to expandable and remove it from available.
Bad Stuff

Rule: When expanding from a location, \textit{cur}, in \textit{expandable} to an adjacent location in \textit{available}, make sure that adding \textit{pos} to \textit{expandable} doesn’t cause any bad stuff.

Opposite:

Opposite kitty-corners:

If \textit{opposite} or either of the \textit{opposite kitty-corners} are in \textit{expandable}, then do not add \textit{pos} to the loop.
Making a Loop cont.

1. Choose a location, *cur* in *expandable* at random.
Making a Loop cont.

1. Choose a location, \textit{cur} in \textit{expandable} at random
2. Look at neighbors to see if and where the loop can expand from \textit{cur}.

2.1 If a neighbor is in \textit{available} and it wasn't a valid neighbor, remove it from \textit{available}
2.2 If there are no valid neighbors, add \textit{cur} to \textit{unexpandable} and remove it from \textit{expandable}
2.3 Otherwise, randomly choose a valid neighbor to add to \textit{expandable} and take out of \textit{available}
3. repeat until there are no locations in \textit{expandable}
Making a Loop cont.

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3. repeat until there are no locations in \textit{expandable}
Making a Loop Example
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[Diagram of a loop example]
Making a Loop Example
Making a Loop Example
Making a Loop Example
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Making a Loop Example
Making a Loop Example
Making a Loop Example
Making a Loop Example
Filling with Numbers

Surprise surprise, this is actually really easy

(sum the differences in the neighboring locations)
Removing Numbers

To make puzzles interesting, we want to remove numbers. We want to do so until we have reached a certain count. Must retain one unique solution.
Removing Numbers

The Process:

1. Pick a number from a set of eligible numbers
2. Add this number to a stack of ineligible numbers
3. Check if eliminating would make the puzzle unsolvable
   3.1 If solvable, remove the number from both the grid and set of eligible numbers
   3.2 If unsolvable, only remove the number from the set of eligible numbers
4. Repeat until set of eligible numbers is empty
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4. Repeat until set of eligible numbers is empty
Removing Numbers cont.

Once set of eligible numbers is empty:

1. Pop numbers off ineligible stack
2. Place each back into the set of eligible numbers
3. Do so until most recently eliminated number is found
4. Keep eliminated in the ineligible stack, but place back into grid

Repeat removing numbers until desired count is reached
Removing Numbers cont.

Once set of eligible numbers is empty:

1. Pop numbers off ineligible stack
Removing Numbers cont.

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Repeat removing numbers until desired count is reached
Removing Numbers cont.

It’s too hard!

Improvements

1. Data
2. Rule set
3. Balancing
As a result, we created two subsets of rules:

- **easy**: the rules with a greater than 5% occurrence rate
- **hard**: the rules with a greater than 1% occurrence rate
An ‘Easy’ Puzzle

```
    . . . .  .  .  .  .  .
    . .  .  .  .  .  2  1  2  2
    2  0  .  .  .  .  3  1  1  1
    . .  .  .  .  .  .  .  .  .
    .  .  .  .  .  .  1  .  .  .
    1  2  3  1  1  .  .  .  .
    .  .  .  .  .  .  1  .  2  .
    .  .  .  .  .  .  .  .  .  .
    1  1  3  1  .  .  .  .  .
```
A Quick Attempt at the ‘Easy’ Puzzle
Balancing the Numbers
A Balanced Easy Puzzle

```
. . . . . . . . . .
  1          2
. . . . . . . . . .
  0  2  2  3  1  3
. . . . . . . . . .
  3  2  2  2  2
. . . . . . . . . .
  3          2
. . . . . . . . . .
  2  3          1  3
. . . . . . . . . .
  2          2  1
. . . . . . . . . .
```
A Quick Attempt at the ‘Easy’ Puzzle
Important details

- The Solver is run on the order of $mn$ times.
- Each time the solver is run, it happens with a maximum depth of one guess which has on the order of $O((mn)^2)$ time.
- Therefore, the generator run in the order of $O((mn)^3)$ time.
References

*Conceptis Puzzles Slitherlink Techniques*

*On the NP-completeness of the Slither Link Puzzle* Takayuki YATO

*Finding All Solutions and Instances of Numberlink and Slitherlink by ZDDs.* Ryo Yoshinaka, Toshiki Saitoh, Jun Kawahara, Koji Tsuruma, Hiroaki Iwashita and Shin-ichi Minato.

*A rule-based approach to the puzzle of Slither Link.* Stefan Herting.


*Solving logical puzzles using mathematical models.* KVIS Susanti, S Lukas.
Thank you
Questions?